Time Series Forecasting in Stock Trading Markets: The Turning Point Curiosity

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Abstract

General Context Univariate Time Series Models [TSM] use only a Panel of historical data to produce forecasts. The tacit belief in using TSM is that the past information portends the future of the longitudinal data-stream. This is likely in certain cases such as strictly Ergodic Panel segments of sufficient size in the overall Panel. A question of interest is: Is the success of TSM in these contexts generalizable? The test of this question used a Litmus-Test design to examine the performance profile of TSM for a longitudinal time series the last point of which is a Turning Point [TP]. Specifically, the inference measure will use the Relative Absolute Error [RAE] of the TSM tested over three forecasting horizons. In this testing, five TSM configurations were employed; the TPs are identified using a fixed screening filter applied to randomly selected firm Panels actively traded on the S&P 500 from 2005 through 2013. There is no evidence that any of the five TSM outperformed the RW model which is incidentally the TP. The impact of these results is that one cannot assume that the effectiveness of TSM generalizes to all domains—in particular—forecasting after TPs that seems to be a Domain Lacuna where the effectiveness of TSM will be compromised.

Key words: Domain Lacuna; Time Series Models; Turning Point; Panels; Random Walk

JEL classification: G13;G17

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Introduction

This introduction first offers a socio-Human Information Processing perspective to rationalize the development of the forecasting context addressed in the research. It is offered that an efficient and effective forecasting model is the Sorcerer’s Stone, the likes of which Harry Potter could only have conjured in his most fanciful moment. Lusk (2019b) notes in a paper that creates forecasting enhancements for use in the Bloomberg™ terminals, in particular, the FA platforms that:

Archimedes remarked: Give Me a Fulcrum, and I Shall Move the World; the simile in the market trading world is; Give Me an Effective Forecasting Model and I can make Bill Gates and Sam Walton look like Paupers.
This motivation and the inherent rewards to “foretell the future” by forecasters: seers: oracles: prophets: soothsayers: or prognosticators is a common thread throughout human history and interestingly seems to transcend culture. Notable in the litany of historical support and the commensurate rewards for divining the future are:

The Bible [King James:DanielC5:V7] The king cried aloud to bring in the astrologers, the Chaldeans, and the soothsayers. And the king spake, and said to the wise men of Babylon, Whosoever shall read this writing, and shew me the interpretation thereof, shall be clothed with scarlet, and have a chain of gold about his neck, and shall be the third ruler in the kingdom (https://www.kingjamesbibleonline.org/Daniel-Chapter-5)

Pythia: the Oracle of Delphi Already, by 480 BCE, the Pythia of Delphi was an ancient institution. The Pythia were the senior priestesses of the Temple of Apollo, the Greek god of prophecy. For more than 1,100 years (until 390 BCE), - - -, they were viewed as the most authoritative soothsayers in Greece. Essay: John C. Hulsman: (https://www.fastcompany.com/40576223/the-ancient-worlds-top-political-consultants-were-all-women)

In Homer's Iliad, Cassandra—[Given the gift of Divination by Apollo who she later rejected and thus Apollo curse her so that no one would a priori believe her previsions which were to come true]—predicted many of the events of the Trojan War. Priam's son Paris planned a trip to Sparta. Cassandra warned against it, but her warnings were ignored. Paris traveled to Sparta, where he kidnapped Helen, starting the war with Greece. Cassandra later predicted Troy's defeat and warned the Trojans not to accept the Greek gift of the Trojan horse. Again she was ignored, and Greek troops hidden inside the wooden horse captured the city (http://www.mythencyclopedia.com/Ca-Cr/Cassandra.html).

The Nechung oracle, the oracle-priest of Tibet who, until the conquest of Tibet in 1959 by the People's Republic of China, was consulted on all important occasions. The priest chosen to be the Nechung oracle was the chief medium of Pe-har, a popular folk divinity incorporated into Buddhism, and resided at the Nechung (Gnas-chung-lcog) monastery near Drepung ('Bras-sprungs), the centre of the Pe-har cult. The oracle is said to have first been appointed government adviser during the time of the fifth Dalai Lama (1617–82). He was required to journey to Lhasa once a year, during the New Year festivities, to prophesy the year’s coming events, and was consulted whenever a search was conducted for a new Dalai Lama. Unofficial visitors were not normally allowed to consult him but were permitted to present questions to him when he was in a trance and after state business had been completed. (https://www.britannica.com/topic/lama#ref95117)

The Magic 8 Ball™ In the U.S., - - -, one of the most-popular such items among kids and adults was the Magic 8 Ball. Other than its being a pivotal and dangerous ball in the billiard game of Eight Ball, however, it may seem a mystery as to why the Magic 8 Ball was the object chosen to be a clairvoyant party favorite. The Magic 8 Ball contains a 20-sided die with 10 positive answers, 5 negative answers, and 5 vague responses such as “Concentrate and ask again” and “Reply hazy, try again.” The answers on the white die are raised so that they can be visible when the die is pressed against the glass. The Magic 8 Ball is now owned by Mattel, Inc., and as of 2012 still sold more than a million units per year (https://www.britannica.com/story/where-did-the-idea-for-the-magic-8-ball-come-from)

These vignettes offer evidence of the human pre-occupation with believing, usually against the common sense of experiential reality, that forecasters that appear to be in a trance-like-state [whether induced by noxious fumes or self-induced as the necessary artistic accoutrement to enable the suspension of dis-belief] provide glimpses into worlds that have yet to happen. If this is the case, and it certainly seems to be, then forecasting models which use real mathematical and statistical platforms and operate in the real Big-Data stock market context are even more likely to garner “devotees” as they offer a dimension of Holdback reality checking and the lure of monetary rewards.
Literature Review

Market Analytics

With such a socio-historical pedigree, it is not in the least surprising that the second event that followed the creation of regulated stock trading markets was the development of forecasting models to ferret-out which stocks would have relatively superior returns. This need-to-know created a forecasting glut that started in the post-WWII era and was fueled by the development of Venture Capital organizations circa 1947, much like the *Shark-Tank™* airing currently on ABC™ now entering its 10th season. Thirty-five years later the glut was a tsunami of biblical proportions. This begged a “reality-check”. Enter Makridakis et al. (1982) and a number of forecasting experts who accepted the challenge to see whose forecasting models could “walk-the-talk”. These Maridakis competitions, and there have been more than a few over the years, have been an invaluable sieve and have culled out a few modeling systems that overall seem to be validated models that beat the forecasting odds more often than not. One class of the models that are regularly a part of these competitions is the non-seasonal/non-cyclical univariate Time Series Models [TSM]. These single variable models use ONLY past information to offer insights to what is likely to happen for a particular longitudinal series past the last observed data-point. There are two important classes of such TSM: (i) Two parameters [Intercept and Slope] linear OLS-Regression models, and (ii) Two parameter [Level and Trend] Exponential Smoothing models that are derived from Moving Average Models. The most effective in this class seems to be the Holt Model [also the ARIMA(0,2,2) model]. These models are often employed in Panels and according to all of the head to head competitions seem to function very well in this context.

Initial CMI Illustration

The question addressed in this paper is: How do these TSM perform in a Panel composed of Ergodic segments that are linked at a point after which there is a contiguous Ergodic segment that if combined with its linked predecessor segment will form a Non-Ergodic Panel? An graphical illustration will be most instructive. Following is the Panel of CMI in the Interval [31March2005 through 28Nov2008]:

![CMI S&P500](image)

**Figure 1:** Panel of CMI S&P 500 Monthly Prices

Figure 1 is the proto-typical example of the Ergodic concept that forms the basis of this research investigation. From Point1 to Point 23 the Panel is trending-UP [OLS:Slope[+3.3; p-value<0.001] and has a stable variation around the trend-line[StDev[4.6]. From Point 24 to Point 45 the Panel is trending-DOWN [OLS:Slope[-4.7; p-value<0.001] and has a stable variation around the trend-line[StDev[19.4]. Clear is that for the first Ergodic Segment [ESI] there is stability in the first two Moments re: the dataset or the residual set; and for the ESI, there is likewise stability in the first two Moments. However, comparatively they differ dramatically in their first two characteristic Moment profiles. These Panels are very typical of Panels of stock prices for Firms
traded on active exchanges most of which test to have Panels that are Fixed Effects in nature. Forecasting will “work” very well for either ESI or ESII. However, any forecast that has a forecasting horizon formed from the TS [1 to 23] that forecasts into the beginning of the second ES: TS [24 to 45] will fail to provide reasonably accurate forecasts as the projection is out of one ES into a differentiated ES that happens to be linked to the first ES. This is akin to a Chaos-Theory transition singularity where in one TS-moment, the market detached from the previous day’s generating process. For example, the market world abruptly changed due to the Lehman Bros™ LLP sub-prime debacle [Circa September 2008].

The intention of offering a brief historical perspective on the “Need to forecast Accurately and the commensurate Rewards” and then to introduce the TSM group of forecasting models sets the following context for this research report:

As the desire to find an accurate forecasting model seems a part of our gnome and TSM are the simplest class of effective forecasting models perchance forecasters are lead to believe the TSM class to be the Forecasting Grail—i.e., they are the universal key to unlocking the future.

These aspects of the forecasting mythos and the results of the Makridakis competitive empirical studies beg an investigation to determine if a Domain-Lacuna exists where TSM are incapable of producing effective forecasts. This seems a reasonable question to motive a study as a search [27March2019] using the ABI-Inform™[ProQuest™] search engine found no information on the Domain restrictions or suggested Lacuna for forecast models of the TSM class. This was surprising and may give the impression that TSM can be expected to “work” without restriction as to the forecasting domain of the problem. In summary, the intention of this research is to determine if there are Domain-Lacuna or restrictions that may help forecasters to make an informed decision in selecting an appropriate forecasting model.

Research and Methodology

Specifically, following is discussed:

1. The definition of the Turing Point [TP] as the Gold Standard construct for examining the information content of economic longitudinal Time Series[TS] and discussion of the implications for forecasting,
2. The selection of the Time Series Models [TSM] used to produce forecasts after the TP using a random sample of firms in the S&P500 Panel from 2005 to 2013,
3. The Relative Absolute Error [RAE] offered as an ideal measure to judge the joint forecasting effectiveness of the TSM and the information content of the longitudinal time series, and
4. A summary of the results and suggested research directions.

Turning Points: The Hardball Construct

For descriptive simplicity, it is assumed that the longitudinal trajectory of a stock price is: An Ergodic Segment [ES] the dynamics of which are characterized by the first two moments. This simply means that an ES has a formal dynamic boundary set over the Panel conditioned by its first two-moments profile. In this context, recalling the CMI example discussion, the TSM should work very well as the Panel is “stable” and, in the main, at the firm level the trajectory of the firm is Fixed Effects rather than Random Effects in nature. In the spirit of this research, the question of interest is: How do TSM fare for a time series characterized by a Panel that is a time stream which has a third variable: an exogenous Shock-variate located anywhere in the Cartesian stream? See Brillinger (1981). This Shock-variable is usually exogenous to the firm and endemic in the market trading world. This shock variable is a particularly sensitive conditioning feature given the Fixed Effects nature of the Panel of firms trading in active exchanges. As mentioned pertaining to the Lehman Bros. example, the “abrupt” change in the trajectory of the Firm’s Panel is often termed a Turning Point.

The Turning Point [TP]

Following on the work of Chen & Chen (2016), the most desirable point in the Panel to forecast is at a TP as it is a point after which there is a major change in trajectory that is enduring for some stochastic period of time. In Brillinger’s nomenclature, the TP is a point of demarcation between the two contiguous ES that are
statistically dissimilar: see Figure 1. Theoretically, this seems to be the best point to launch a forecast because if one is aware that the last observation of the ES is a TP, and uses this information to condition the forecasting model, this conditioned forecasting model will likely perform well in forecasting into the contiguous ES. This is the issue addressed in this research: If the conditioning information were to be endemic to the Panel and the TSM could have made the needed adjustment, the forecasting model should be effective. However, if for whatever reason, this is not the case then other conditioning variables are likely needed—i.e., a multivariable TSM is required. If this is the case, then this would constitute a Domain Lacuna for univariate TSM.

Design of the TSM Test

The testing construct needed to investigate the above condition effect is a filter that will identify or flag the break-point in the ES. Also see Nyberg (2013, p. 3352). In this regard, Lusk (2018) selected the following measure, SRC, which will be used in this paper. Lusk (2018) offers that the SRC is: relevant, reliable, and independent—non-conditioned on other Market Navigation Parameters—and so is a reasonable measure of the change of a stock price valued at the bell-price in the univariate context:

$$\text{Signed Relative Change [SRC]} = \sum_{t} \left( \frac{Y_{t+1} - Y_{t}}{Y_{t}} \right)$$

where: $Y_{t}$ is monthly average reported by WRDS™ for the S&P500 at month, $t$, $n=4$, $i=1,2,3,4$.

Additionally, as does Chen & Chen, it is necessary for a screening protocol to identify an important change in trajectory in the stock trading Panel; to this end a Dramatic TP is recorded if the Abs[SRC] > 25% and is noted as a DTP. As the DTP is, prima facie, the critical forecasting point in a firm’s Panel, this begs the central question: Does a DTP preclude the class of univariate time series model from inclusion in the panoply of the forecaster?

Forecasting in DTP Environments

The only citation that was retrieved regarding the “logical possible disconnect” in forecasting in DTP environments was offered by Larson (2011):

Finally, I explore the ability of simple time series models to forecast regional house price dynamics. I find that theory-driven multivariate models were best able to forecast the declines in house prices experienced in California from 2007-2009. Univariate, atheoretical models, on the other hand, forecasted quite poorly and were unable to detect turning points in the housing market.

Interestingly, there were no citations in the peer reported literature of the “logical possible disconnect” in using univariate TSM in forecasting into horizons after the DTP; however, there were a number of interesting forecasting model manipulations of the Panel information set to anticipate a DTP. Simple models offered by Bhandari (2012 & 2017), a must read for any student in a market trading course, offer VBA projection possibilities. The Bhandari models use three Panel values: (i) the previous day High, (ii) the previous day Low, and (iii) the final Bell-Price close to form “Support and Resistance” boundaries in the “break-through” inferential class. The Bhandari models are in the class of Charting Models that have been used from the inception of trading markets. While such models do use the Panel data leading to a “Pivot-Point”, they do not quality as a traditional univariate TSM. A derivative class of charting models is the Neural-Net models; Neural is a label that indicates learning from past experience and so offer a dynamic feature of Panel encoding where there can be adaptive behavior. See Lee & Tzeng (2013) who applied these adaptive re-configurations to focus on the DTP as the focal event.

Configuration of the TSM Context

However interesting the Charting and Learning models may be, they are not in the focus of this paper. The question of interest addresses the rapport between the DTP and univariate TSM relative to forecasting acuity and possible Domain Lacuna. As the central feature of this montage is the SRC, it will be useful to examine in detail the nature of this SRC-screen. The SRC is a simple short-term Smoothing filter in the Mean-Class. In this case, given the expected stochastic variation in an auto-correlated environment characterized by Fixed
Effects, it is expected that the longer the screening filter the more DTPs will be identified/flagged using EQ1. And, by symmetry, the shorter the Smoothing filter the less DTPs will be identified. For example, for the stock CUMMINS INC, [CMI] over the S&P500 Panel from 2005 through 2013, the SRC flags 17.3% of the months as DTPs over the rolling S&P500 Panel. If one doubles the SRC-the Screen to eight months, the percentage of SRC-flags goes to 27.6% a 59.5% Increase. If one reduces SRC-Screen by 50%, the number of DTPs flagged is 11.2%, a reduction of 35.3%. In the case of calibrating the SRC, one seeks a balance. As the decision-maker will need to use the DTP information to effect action plans, a four month waiting period seems to be in the “Goldilocks Zone”: Not too long: Not too short: Just right. Therefore, the Lusk (2018) calibration as scripted in SRC: EQ1 seems reasonable. However, to be clear: The definition of a SRC fixes the DTP in the past relative to the current set of data. To this extent, this is NOT likely to be a practical construct in the dynamic market trading world. This is NOT a problem as the more basic question is posed:

*What-If the DM were to have flagged a particular month as a DTP—ignoring for the moment HOW the DM would actually effect such an identification? IF the DM were to know a month to be a DTP, are there univariate TSMs that would be useful in creating an effective forecast of the likely S&P500 values for periods after the DTP? IF so, then this would rationalize these TSM as a reasonable choice-set if ES transitions would be likely in the Panel.*

### An Illustration

A detailed example will aid in elucidating these protocols. Consider the case of: CUMMINS INC. [CMI:CMMNP designation]. The S&P500 Panel where, using EQ1, a DTP was located from the following dataset:

<table>
<thead>
<tr>
<th>CMI: S&amp;P500 Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>110.21</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>119.23</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>119.92</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>92.16</td>
</tr>
<tr>
<td>94.23</td>
</tr>
<tr>
<td>101.21</td>
</tr>
<tr>
<td>118.70</td>
</tr>
</tbody>
</table>

Following are the n = 10 SRC values that are produced from EQ1:

<table>
<thead>
<tr>
<th>SRC Computation for CMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.073632</td>
</tr>
<tr>
<td>-0.02243</td>
</tr>
<tr>
<td>0.027671</td>
</tr>
<tr>
<td>0.090163</td>
</tr>
<tr>
<td>0.082068</td>
</tr>
<tr>
<td>0.050953</td>
</tr>
<tr>
<td>0.054953</td>
</tr>
<tr>
<td>-0.13475</td>
</tr>
<tr>
<td>-0.19703</td>
</tr>
<tr>
<td>-0.29813</td>
</tr>
<tr>
<td>N/A</td>
</tr>
</tbody>
</table>

For example, consider the identification of the DTP: 144.72.

\[-29.813\% = \frac{[\text{Average}[92.16, 94.23, 101.21, 118.70] - 144.72]}{144.72}\]

The Point 144.72 is a *Dramatic TP [DTP]* as at 144.72 is where EQ1 produces an ABS[-0.2913] > 25%.

### The Set of Time Series Models Used in the Study

The context for this research is to determine if a longitudinal time-stream of data anchored by a DTP and filtered by a univariate OLS-two-parameter[Intercept & Slope] TS-Forecasting Model will exhibit forecasting acuity—that is, produce useful forecasts. In this case, there are three test-contextual Nulls: (i) The Signal Information in the time stream will NOT indicate an impending change after the DTP, OR (ii) the TSM is NOT capable of detecting the correct signal embedded in the Panel, OR (iii) even if (ii) were not to be the case, the TSM is NOT an appropriate model compliment to the longitudinal time-stream of data. As these are OR conditioned inferential Nulls failing to reject any versions of these Nulls will tacitly suggest a Domain issue for the forecasting model. However, as there will only be one testing frame the three conditions are jointly implied in the inferential testing context. Simply, failing to reject the Null of forecasting acuity will mean either: (i) there is NO signaling information, OR (ii) the univariate TSM was not capable of detecting the signal, OR (iii) the TSM was capable of detecting the signal BUT the Domain into which the forecasts were projected was not in the assumption set of the TSM. This joint result will be informative and is implied in the focus of the study.

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The Standard OLS [Intercept & Slope]

Linear univariate Time-Series Forecasting Model [OLSR]. This is the model that has the currency of precedent. It was used in the CAPM studies by all four of the “inventors” of the CAPM model: Treynor(1962), Sharpe(1964), Lintner(1965) & Mossin(1966) who collectively detailed the valuable information of a longitudinal time-stream of data of firm stock-trading values regressed against the market, usually the S&P500. In this context the principal interest was in the volatility filter of the slope or β. Also, subsequent research by Fama & French (1992 & 2012) continued to use this model form with a series of blocking modifications. The fact that the OLSR model form was used in this important search, that garnered a Noble Prize in Economics, establishes the credibility of the OLSR as a reasonable model choice. Finally, certainly adding to the credibility of the OLSR model it was also used in the various M-competitions that started with Makridakis et al. (1982), and was later used by Collopy and Armstrong in their groundbreaking research termed Rule Based Forecasting (1992).

The OLSR model formed/fitted using N Panel points is:

\[ \hat{y}_{(t=(\Omega))} = \hat{\alpha} + \hat{\beta} \times [\Omega]; \quad \Omega: 1, 2, \ldots, n \]  
(2)

Where: N is the number of points in the Firm Panel, the last point used to fit the model is N that is also the DTP, and \( \Omega \) is the forecasting time index that ends at the nth point.

The ARIMA(0,2,2)/Holt

While there is a very strong precedent for the OLSR model from a technical perspective, in the context of the longitudinal time-stream of data, the OLSR filter usually does not remove strong autocorrelation from the longitudinal time-stream of data; and, in some cases it increases it! This is a problem in that the assumption of the OLSR filter is that the residual series output of the filter is White Noise—i.e., the Null of the Autocorrelation test is the state of nature for the residuals. For this reason, the Holt model was one of the basic models in the Makridakis et al. (1982) competition and was later used by Collopy and Armstrong (1992) in their Rule Based Forecasting Model. The model form for the Holt model is rather complicated and may be found at the following URL. In this study, the Holt model used is found in the SAS:Platform. The great advantage of this SAS-platform is that the initializing parameters {α & β} of the Holt Model are optimized for the given dataset and Stability and Invertability information is reported.

The Rule Based Forecasting Configuration

For an historical context the Rule Based Forecasting [RBF] model of Collopy and Armstrong (1992) derived much of its technical montage from the Makridakis et al. (1982) M-Competition. The initial roll-out of the RBF expert system focused on the Random Walk [Naïve I], the OLSR, Browns, and the ARIMA(0,2,2)/Holt. As Collopy & Armstrong (1992) note in the summation of their research report:

- The rule-based forecasting procedure offers promise. We provide our rules as a starting point. Hopefully, they will be replaced by simpler and fewer rules.[ p.1408]

This is what has happened over the years. Most of the recalibration was done by: Adya (2000) and Adya, Collopy, Armstrong & Kennedy (2001). Effectively, this recalibration research eliminated the Browns Model and re-tooled the four sets of initial judgmental weights. This left the Random Walk—the last point in the Panel, the OLSR, and the ARIMA(0,2,2)/Holt as the basic model set for the RBF model. As the projection for the forecasting models are into the short term horizon—{(Hor1, Hor2 & Hor3)} a blend of the various model weights reported over the decades since the initial roll-out of the RBF model were used. Specifically, the forecasts from the Random Walk, the OLSR, and the Holt will be weighted: {40%, 20% & 40%} respectively. See Adya & Lusk (2013). This is consistent with the initial values offered by Collopy and Armstrong in that in the short run—in this case three periods ahead—the weight given to the temporal anchoring value of the Random Walk and the Holt that is sensitive to short-term directional changes are both given twice the average weight as the long-term trend formed from the OLSR. Additionally, Collopy and Armstrong (1992)
experimented with an equal weighting protocol of their four basic models and it was found to also provide reasonable forecasts. For example, they note:

- Combining has performed well in published empirical comparisons (Clemen 1989). Equal-weights combining has generally been more accurate than the average accuracy of the individual forecasts used in the combination. Sometimes it has been more accurate than the best of the individual forecasts. - - - Equal-weights combining yielded substantial gains over typical method-five for one-year-ahead forecasts, as its MdAPE was 22% lower. [p.1404]

Additionally,

- For series with much instability (two or more instability features), we did not expect rule-based forecasting to be superior to equal weights or the random walk. In fact, it was a bit more accurate than the random walk, but slightly less accurate than equal-weights. When only one instability feature was present, rule-based forecasting was significantly more accurate than equal-weights (p = 0.01 using a one-tail Wilcoxon test of the CumRAE). A similar superiority was found when the series had no instabilities. [p.1406]

Forecasting Models Employed

Therefore, in this research report the RBF weights as well as the RBF equal weights will be used. This produces the following model set which will be used to generate forecasts:

1. Univariate OLS [Intercept & Slope] Linear Regression noted as: [OLSR],
2. Holt or ARIMA(0,2,2) [Level & Trend] Exponential Smoothing noted as: [Holt],
3. RBF: Standard [RW × 40% + OLSR × 20% + Holt × 40%] noted as: [RBF],
4. RBF: Equal Weights: [(RW + OLSR + Holt)/3]: noted as: [RBF:EW]
5. Overall Equal Weights: [(OLSR + Holt + RBF + RBF:EW)/4] noted as: [OEW].

These five models will be benchmarked by the Relative Absolute Error [RAE] using the RW model. Consider the logic of this benchmark.

The Relative Absolute Error

Accepting that a TSM must provide an inferential advantage relative to an alternative projecting protocol the first issue is to select a benchmark for testing the TSM-set. The benchmark that has achieved currency is the Random Walk [RW] or the Naïve I—using the last value in the time stream as the forecast over all the forecasting horizons. In the Makridakis et al. (1982) M-competition this RW-value provided competitive forecasts compared to many of the other models that were tested in the M-competition. In fact, the forecast acuity of the RW-value, the most simple projection, led Collopy and Armstrong (1992) to select it as one of the basic models in their groundbreaking RBF Expert System Model. Given the performance of the RW-value it is a perfect benchmark for any forecast and so has achieved currency in the forecasting milieu. Thus in this study the TS forecasts will be benchmarked by the RAE defined as:

For an assumed Panel of ten (10) of the S&P500 values, the last of which is time indexed as: \( Y_{t=10} \) that is also the Turning Point as well as the RW, a forecasting model \( f() \), and a one-period-ahead forecast of the S&P500, noted as \( \hat{Y}_{t+1} \), the Relative Absolute Error [RAE] is:

\[
RAE[\hat{Y}_{t+1}] = \frac{ABS[\hat{Y}_{t+1} - A_{t+1}]}{ABS[Y_{t=10} - A_{t+1}]} \tag{3}
\]

Where: ABS is the absolute value operator, \( A_{t+1} \) is the designation for the Actual value in the S&P500 Panel at time \( t+1 \); \( Y_{t=10} \) is the Turning Point—i.e., the S&P500 Panel value at \( t=10 \) which is also the RW-value.

The logic of using the RAE as a measure of forecasting acuity is intuitive. It simply says that if the RAE is \( =1.0\), the forecast error of using the DTP as the one-period-ahead forecast—i.e., the RW value, gives the same forecasting error as does the forecasting model. If the RAE is \( >1.0\), it indicates that the DTP:RW as
the forecast outperforms the forecasting model. Finally, if the RAE is <0, the forecasting model is better than using the DTP: RW as the forecast. In the first two cases, one would reject the forecasting model and just use the "Occam's Razor" model: the DTP. At this point, an illustrative example will aid the exposition.

**Full-Model Illustration: CMI**

In this case, assume that we have the S&P500 dataset, n=10, presented in Table 1. This Panel for the CMI dataset as identified using the SRC EQ1 generated the following five model values: Table 3

<table>
<thead>
<tr>
<th>Horizons</th>
<th>RW[Naïve1]</th>
<th>OLSR</th>
<th>Holt</th>
<th>RBF</th>
<th>RBF:EW</th>
<th>OEW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hor1</td>
<td>144.72</td>
<td>141.3587</td>
<td>148.5422</td>
<td>145.5766</td>
<td>144.8736</td>
<td>145.0142</td>
</tr>
<tr>
<td>Hor2</td>
<td>144.72</td>
<td>144.4368</td>
<td>155.9515</td>
<td>149.1560</td>
<td>148.3694</td>
<td>148.5267</td>
</tr>
<tr>
<td>Hor3</td>
<td>144.72</td>
<td>147.5149</td>
<td>163.3608</td>
<td>152.7353</td>
<td>151.8652</td>
<td>152.0393</td>
</tr>
</tbody>
</table>

**Computational Base To enrich the exposition all of these computations will be illustrated for the second forecasting horizon [H2].**

DTP:RW[Naïve1]: The actual turning point flagged using EQ1 is 144.72. As this is the last observed data point this is by definition, the DTP:RW value.

OLSR[Following Equation 2]:144.4368 = [107.4993 + 3.0781 × (10 +2)]


RBF: 149.1560 = [(144.72 × 40%) + (144.4368 × 20%) + (155.9515 × 40%)]

RBF:EW 148.3694 = [AVERAGE[144.72; 144.4368; 155.9515.]

OEW: 148.5267 = [AVERAGE[144.72; 144.4368; 155.9515; 149.1560; 148.3694 ]]

These are the five forecasts for the three periods {Hor1; Hor2; Hor3}. However, to adequately profile the nature of these forecasts a benchmark would be of great value. As discussed above an ideal measure that has achieve its due currency is the RAE. The forecasts in Table 3 are reported in the RAE measure as follows:

<table>
<thead>
<tr>
<th>Horizons</th>
<th>OLSR</th>
<th>Holt</th>
<th>RBF</th>
<th>RBF:EW</th>
<th>O:EW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hor1</td>
<td>0.936048</td>
<td>1.072721</td>
<td>1.016298</td>
<td>1.002922</td>
<td>1.005597</td>
</tr>
<tr>
<td>Hor2</td>
<td>0.994391</td>
<td>1.22245</td>
<td>1.087859</td>
<td>1.07228</td>
<td>1.075395</td>
</tr>
<tr>
<td>Hor3</td>
<td>1.064236</td>
<td>1.428426</td>
<td>1.184217</td>
<td>1.16422</td>
<td>1.168221</td>
</tr>
</tbody>
</table>

**Exploratory Caveat** As the DTP and so the RW variable is identified as the last value before a dramatic change in the trajectory of a stock—a point of a precipice in the case of an impending dramatic change in the trajectory of a stock—one may conjecture that the SRC protocol is bias to produce very large RAE values. To address this anecdotally, a set of data was randomly sampled from the Bed Bath & Beyond™ [BBBY] using the Bloomberg Terminals. The sampled data was a Panel from T1[2005] to 2018. This data exhibits a statistically significant Pearson Autocorrelation and so the Holt model is the model of choice; the RAE Holt: profiles are presented in Table 5.
The point of using the BBBY data sample is to demonstrate that the SRC measure is NOT likely biased to producing RAE values that are relatively large that would confound the test of the research questions where the RAE is the test-effect measure. Using the standard parametric ANOVA with the Welch-correction as well as the Wilcoxon and the Median Tests from the SAS.JMP.v13 platform as a robustness check test, the non-Directional Null was not rejected (the smallest p-value of the three tests was: >0.25;—i.e., recognizing the Power caveat, there is no indication that the central tendency of the SRC RAE profile of Table 4 is different than that of the test set in Table 5. Further, the BBBY dataset actually has an average that is consistently either not different or in the wrong direction even using the Armstrong & Collopy (1992) RAE-correction of [0.01 & 10.0]. The inference offered is that there is no evidence from this illustrative example that there is a confounding effect-bias in using EQ1 to identify DTP segments. This being the case, following is the analysis and the inference protocol to address the question of testing forecasts after the identification of a DTP using five Time Series models presented above.

Testing Protocol: DTP Forecasting

The a priori Hypothesis [alternative form] to be tested is:

- Ha Given that the five TS models use contiguous Panel information indexed back from the DTP, and given that the DTP is the RW that is only one data point of that Panel, the TS models are expected to provide effective forecasts characterized by RAEs lower than 1.0 over the three forecasting horizons.

Rationale: This simply asserts that any forecasting model that uses a Panel of data should/will outperform a forecasting model that used only one data point in the Panel—albeit the last. The statistical justification is immediate as the precision of the Confidence Interval estimates is an inverse function of the sample size. Given this, the critical alternative is: IF Ha is not the case that would likely strongly suggest that the forecasting domain is NOT in the assumption set of the TSM used.

The profile of the Medians: Means of the accrual firms are presented in Table 6:
Table 6: RAE Profile of the Forecasting Models over the Horizons

<table>
<thead>
<tr>
<th></th>
<th>O:EW</th>
<th>RBF:EW</th>
<th>Holt</th>
<th>OLSR</th>
<th>RBF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hor1</td>
<td>1.03:0.98</td>
<td>1.03:0.99</td>
<td>0.98:0.96</td>
<td>1.07:1.09</td>
<td>1.02:0.97</td>
</tr>
<tr>
<td>Hor2</td>
<td>1.06:1.05</td>
<td>1.06:1.06</td>
<td>1.01:1.11</td>
<td>1.09:1.13</td>
<td>1.04:1.04</td>
</tr>
<tr>
<td>Hor3</td>
<td>1.08:1.32</td>
<td>1.09:1.32</td>
<td>1.04:1.25</td>
<td>1.10:1.46</td>
<td>1.05:1.30</td>
</tr>
</tbody>
</table>

Validation Vetting of the Accrual Set

Before the inference testing of Ho/Ha is effected and discussed, it is instructive to offer a validation or contextual testing that will impact the nature of the inference derived from the testing of Ha relative to Ho. This is sometimes referred to as vetting the accrual set upon which the principal tests are formed. If there are expectations that are related to “General Knowledge” that underlie the nature of the experimental design, then these insights can be used to reject the Random accrual condition that would cast doubt on the generalization of the results of the testing of Ho/Ha. Simply, if one finds that the random accrual of the datasets happened to be a-typical, then this would create an inferential caution that the results may not be generalizable. Consider the vetting tests for examining the generalizability of the results.

Vetting Test I

It is axiomatic and time-tested that forecast accuracy is an inverse function of the number of periods-ahead forecasted. Simply, the longer the forecasting horizon the higher the forecasting error. This being the case, it would be inferentially troubling if the RAEs were found NOT to be increasing over the three forecasting horizons. Table 6 provides a clear binary check. The vetting expectation is thus: It is expected that the RAE-Means or the RAE-Medians of Table 6 would be ordered as follows: Hor1 < those of Hor2 and Hor2 < those of Hor3 and by transitivity: Hor1 < those of Hor3 for each the five models. Conservatively (i) using either the Mean or the Median values, and (ii) ignoring the transitivity check, the p-value for this binary association assuming that the horizons are not associated directionally would be: 0.01%: (50%)^10. This ordered profile is consistent with the general expectation of decreasing forecast accuracy of the three longitudinal projections. The Binary p-value suggests that this ordering would occur 1 time in 10,000 when the Null is the State of Nature. This strong rejection of the Null should allay any reasonable concern that the forecast projections are not consistent with the inverse: Accuracy v. Horizon profile.

Vetting II: Power Context

For Ho/Ha As there are a number of possible Model & Horizon design features, each one of which has a number of Power parameters, a simple “passpartous” or generic Power calculation has been selected. In this Power context, it is instructive to re-define and so reverse the testing logic. The simplest historical expectation is that the RAE-profile for univariate TSMs is 85%. This is conservative, as the work of Armstrong & Collopy (1992) and Adya, Lusk & Belhadjali (2009) found in many forecasting contexts that the RAE for RBF-TS models was in the practical range of [55% to 75%]. Thus, assuming that the historical profile expectation is Ho=0.85, then a reasonable test-against alternative is that the TSMs do not outperform the RW, in which case, the RAE would be equal to 1.0 or Ha = 1.0. In this case, the test is to determine if the historical state of nature, Ho, [the RAE is 85%] is consistent with the testing relative to the design circumstance of this research. Therefore, a creative Power test will be to calculate how often does one fail to reject Ho if the worrisome actual state of nature is Ha. This vetting inference asks: When the actual effect is that the forecasting error of the RW is no different from that of the TSMs how often do we fail to reject the historical perspective of 85% as the RAE when the alternative Ha is the true state of nature; this is the False Negative Error [FNE] for this context; simply one believes that the historical perceptive remains to be the case when the reality is that it is not the case and the RAE is 1.0. If this happens rarely then this gives confidence that rarely will one believe that the RAE is <1.0 when in fact this it is not the case. In forming the Power [1-FNE] for the overall analyses, the test will use the historical perspective as the test of the “persistent” state of nature—i.e., the RAE for the models is Ho=0.85. Using this as the context for the FNE test, the test against value of Ha=1.0, and using the standard error [Se = 0.0456] that was the ex-post Se of the 480 RAE values over the three horizons, this gives the FPE[α-point] of: 0.956438: [0.85 + 2.3342×0.0456]; where: 2.3342 gives a directional t-FPE of 99.0%: T.INV(99%,479). In this case, the FNE using: [(0.956438 – 1.0)/(0.0456)]
there are individual Means that are directionally dissimilar relative to the others. There is no evidence at a screening level of 5% that among the ten Mean pair HSD test is an exact over the means [Largest & Smallest] to calibrate the relative total CC. The Tukey Honestly Significant Difference [HSD] test is 

The Tukey: HSD Test [α:0.05] Hor1[None];Hor2[None];Hor3[None] others.

levels of the five models that there are subsets of these Means that are directionally dissimilar relative to the others.

For each of the Horizons, n=160, there is no evidence that trimming end-point values of [0.01 & 10.0] if the RAE <0.01 or the RAE > 10]. With this calibration the Mean was 1.137 and the Se was 0.0456. In this case, the 95%CI were: 1.05 & 1.23. Further, using the value of Ho = 85%, the p-value for the directional test was < 0.0001. Meaning the chance of finding a mean of 1.137 when the historical RAE were to have been 0.85 would occur by chance less than 1 time in 10,000 trials. This is not a sufficiently convincing probability to fail to reject Ho and thus incorrectly inferring that Ha is the not the likely case.

The information gleaned from these three accrual vetting trials is clear: The 16 Firms randomly accrued, as noted in the Appendix, fit the post-hoc testing expectations for firms from which generalizations would be permitted. They: (i) exhibit the classical Accuracy & Error inversion for the number of time periods forecasted into the future, (ii) have a number of DTPs as are expected to occur for firms actively trading in exchanges[detailed following], and (iii) recalling the discussion of the number of DTPs relative to the length of the screening filter in a Fixed Effects context, that have as expected, Fixed Effect profiles that, enable inferential precision for the RAE testing protocol—i.e., the sample size should be able to detect a model-RAE that differs from the historical expectation. Simply, these firms do not seem to be variant or a-typical firms in the market trading context and thus this gives assurance relative to generalizability of the results. Consider now the test profiles use to produce the inference information for Ha.

Inferential Tests

There were 480 values over the three forecasting horizons. We used the Armstrong and Colpophy (1992) RAE trimming end-point values of [0.01 & 10.0] if the RAE <0.01 or the RAE > 10]. With this calibration the Mean was 1.137 and the Se was 0.0456. In this case, the 95%CI were: 1.05 & 1.23. Further, using the value of Ho = 85%, the p-value for the directional test was < 0.0001. Meaning the chance of finding a mean of 1.137 when the historical RAE were to have been 0.85 would occur by chance less than 1 time in 10,000 trials. This is not a sufficiently convincing probability to fail to reject Ho and thus incorrectly inferring that Ha is the not the likely case.

The information gleaned from these three accrual vetting trials is clear: The 16 Firms randomly accrued, as noted in the Appendix, fit the post-hoc testing expectations for firms from which generalizations would be permitted. They: (i) exhibit the classical Accuracy & Error inversion for the number of time periods forecasted into the future, (ii) have a number of DTPs as are expected to occur for firms actively trading in exchanges[detailed following], and (iii) recalling the discussion of the number of DTPs relative to the length of the screening filter in a Fixed Effects context, that have as expected, Fixed Effect profiles that, enable inferential precision for the RAE testing protocol—i.e., the sample size should be able to detect a model-RAE that differs from the historical expectation. Simply, these firms do not seem to be variant or a-typical firms in the market trading context and thus this gives assurance relative to generalizability of the results. Consider now the test profiles use to produce the inference information for Ha.

Inferential Tests

There were 16 firms that were randomly selected from the S&P500 that had full S&P500 Panels from 2005 through 2013; overall they had 32 SRC dramatic changes. It is important to note that on average there are two DTPs for each of the accrual firms. This suggests strongly that DTPs are NOT rare events and so consistent with expected Fixed Effects ES distributed over the Panel rationalizing that the testing issues addressed in this research are important in providing guidance to forecasters re: Domain Restrictions.

Each of the 15 Data Sets represented in Table 6 has 32 observations for a total sample size of 480 [32 x 15]. To give a context for the inferential testing it is noteworthy that all of the 95%CIs for the Model means for each of the 15 sets of data in Table 6 contained 1.0 [n =75]. To further investigate the data of Table 6 the following analyses were conducted. All of these tests are found in: SAS™[JMP™v.13: Analysis;FitYbyX].

ParametricANOVA:Welch

Excepting Hor3 there were indications that for the five models there were variance differences at a p-value level <0.1. Therefore, separate rather than the pooled variance model [due to Welch] was used for inference for all three horizons. Inference: For each of the Horizons, n=160, there is no evidence that among the Mean levels of the five models that there are subsets of these Means that are directionally dissimilar relative to the others.

The Tukey: HSD Test [α:0.05] Hor1[None];Hor2[None];Hor3[None]

The Tukey Honestly Significant Difference [HSD] test is an overall post-hoc test that uses the actual distance over the means [Largest & Smallest] to calibrate the relative total CC-distance of the model RAE-Means. The HSD test is an exact-FPE test re: Table 6 as the sample sizes are equal. Inference: For each of the Horizons there is no evidence, at a screening level of 5%, that among the ten Mean pair-wise comparisons [C^2_{ij}] that there are individual Means that are directionally dissimilar relative to the others.
Kruskal/Wallis Test [Rank-Sum]

This test uses the Ranks of the pooled data and then examines the distribution of the Ranks blocked by the Models. **Inference:** For each of the Horizons, there is no evidence that the model populations from which the sample were taken are sufficiently dissimilar to be identified as not belonging to the same population set given under the general assumption of the test.

Median Test

This test uses a binary [0:Lower]:1[NotLower] scoring individually assigned to the overall Ranks of the RAE-scores. **Inference:** For each of the Horizons, there is no evidence that there is a structural central tendency displacement among the five models after accounting for chance.

Wilcoxon Pair-wise MCTs

This test computes p-values for each of the ten Mean pair-wise Differences \([C^2]\) for each of the Horizons. **Inference:** For each of the Horizons, there is no evidence that among the Mean levels of the five models that there are subsets of these Means that are directionally dis-similar relative to the others. No p-value among the 30 tests was < 0.25.

Summary of the Above Testing

Accepting that the set of Time-Series models employed in this study are the standard models used in most of the time series studies reported in the literature—thus vetted by precedence—the critical overall inference of this study is:

- The univariate TSM used in this study, however effective they may have been in most of the forecasting studies reported over the last 50 years, they do not perform well in forecasting into post-DTP ES. **Summary:** There seems to be a Domain Lacuna for such TSM when one anticipates the occurrence of DTPs as there are in the stock market trading context.

Conclusions

Considering this summary inferential information, the following advice regarding the above *Forecasting Domain-Lacuna* is offered to forecasters:

If, in the Panel, one anticipates that there are likely to be DTPs linking the ES, it would be recommended to enrich the forecasting model by adding conditioning X-variates suitable for Multi-variant models such as:

1. (i) Transfer-Function-Models in the very rich ARIMA class,
2. (ii) Charting or Single-Panel re-configurations, or

In these cases, the selection criterial for choosing variables for the Multi-variate model forecasting into Panels where DTPs can arise should be X-variables that are NOT positively associated with the Y-series to be forecasted as they will be conditioned in a way that inhibits effective forecasting. This is just another way of expressing the essential assumption of forecasting in the market trading world: There needs to be independent generating functions as drivers of the forecasting model. This result was essentially that reported in a recent study of the CapitalCube™ market navigation platform. See [Lusk (2019a)].

For an illustrative example, consider expanding the TS forecasting model class to the Y:X class. This will illustrate the impact result of this study where, like the results reported by Larson (2011), it is the case that univariate TSM are ineffective due to a Domain Lacuna. Assume that the CMI dataset is bi-partitioned as: Y:[Point 2 through Point 23]:ESI n=22 and a Matched Panel for an OLS Y:X-regression with the Dataset: X:[Point 24 to Point 45]:ESII n=22. The OLS TS projections relative to the Actual Values and the OLS Y:X model are presented in Table 7:
Table 7: The RAE profile for an Y:X Model v. the OLS Model

<table>
<thead>
<tr>
<th></th>
<th>RW</th>
<th>TS:ES1</th>
<th>Actual values</th>
<th>Y:SES2</th>
<th>RAE[TS]</th>
<th>RAE[Y:X]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hor1</td>
<td>144.72</td>
<td>142.8908</td>
<td>92.16</td>
<td>130.5970</td>
<td>0.965198</td>
<td>0.731330</td>
</tr>
<tr>
<td>Hor2</td>
<td>144.72</td>
<td>146.2082</td>
<td>94.23</td>
<td>126.7189</td>
<td>1.029474</td>
<td>0.643449</td>
</tr>
<tr>
<td>Hor3</td>
<td>144.72</td>
<td>149.5255</td>
<td>101.21</td>
<td>123.7421</td>
<td>1.110447</td>
<td>0.517807</td>
</tr>
</tbody>
</table>

In this illustrative case, the OLS:Y:X projection model of ES2 is: Y = 139.968 - [0.5063 × [X:Projection ix]] can be compared to the OLSR Time Series model form ES1: Y = 66.591 + 3.317 × [TS:Projection]. It is clear from this example that the use of an adequate X-bench mark that is off-the-indexed positive rhythm of the Y-variate [The Correlation[Y:X]] is -0.83] will aid in the forecasting projection into the DTP-Domain. In this case, the RAE[Y:X] of ES2 is on the average with those reported in the RBF context [Average[0.63]] while the RAE[TS], which is not conditioned to anticipate the which Domain Lacuna, is in the 1.0 range[Average[1.04]].

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References


Appendix

Table: S&P500 Ticker Symbols of the Accrual Firms

<table>
<thead>
<tr>
<th>CMI</th>
<th>HON</th>
<th>DOV</th>
<th>ETN</th>
<th>GD</th>
<th>GE</th>
<th>TXT</th>
<th>NOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLS</td>
<td>PH</td>
<td>PBI</td>
<td>MCO</td>
<td>SNA</td>
<td>PWR</td>
<td>CSX</td>
<td>R</td>
</tr>
</tbody>
</table>

Lusk & Heilig (2019c) report that for a random sample from the S&P500 of 75 Firm-Panel arrangements there were only 12% that exhibited Random Effects profiles—Failed to reject the Hausman-Null. This suggests that in the practical case that there are Hausman-Fixed Effects as the characterization of traded firms.

As a clarification, theoretically an Ergodic Segment [ES] is a subset of a stationary process.

An excellent review of Charting Software is found at: https://www.liberatedstocktrader.com/top-10-best-stock-market-analysis-software-review/. In addition, recommended as an elaboration of the Bhandari Model of 2012 are: (i) the Stochastic Oscillator model of Bhandari (2017), and (ii) in the same vein, the Harmonic Model rendering of Duddella (2017).

In this study, the TPs were not prescreened to eliminate any of them that did not fit the Panel Correlation screen or the Mean test screen over the Pre- & Post-DTP sub-panel segments as used in Lusk (2018). Lusk (2018) did this screening which was effectively a bias to rejecting the Null. However, for the forecasting context in this paper such a pre-conditioning or screening will be relaxed so as to eliminate any related bias to accentuate the Lacuna-effect.


The Power Calculation [1-FNE] is: \( \Delta = [t_{FPE}[\alpha] - \left( \frac{|\text{Abs}[\text{Ho-Ha}|]}{\text{Se}} \right)] \rightarrow [1-FNE] = T.DIST.RT(\Delta, 479) \rightarrow T.DIST.RT((2.3342 - 3.2895), 479): 83.0\%. In this context, one sees that it is possible to switch the valuation of Ho with Ha as the ABS will correct for this assuming that the Se is not affected.

In this case, as the generalizability of the Ha/Ho results seem founded this would suggest that the standard p-values reported under estimate the strength of the rejection of the Null.

In this case, so as not to bias the results to failing to reject the Null, the X-variate projections were created by the ARIMA (0,2,2) Model. These values are: \( Y[23] = 18.51; Y[24] = 26.17; Y[25] = 32.06 \)