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Forecasting with ARMA models: A case study of the exchange rate between the US Dollar and a unit of the British Pound

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ABSTRACT

This paper employs the R software in identifying the most suitable ARMA model for forecasting the growth rate of the exchange rate between the US dollar and a unit of the British pound. The data is systematically split into two distinct periods identified as the in-sample period and the out of sample period. The best model selected for the in-sample period is used to make forecasts for the out of sample period. Both traditional and rolling window forecasting methods are employed. This research uses the MSE, MAE, MAPE and correct sign prediction criterion to compare the forecasting performance of the rolling window forecasting method and the traditional forecasting method. The results obtained suggest that the traditional forecasting method performs better judging by MSE, MAE and MAPE. In other words, the traditional forecasting method is more suitable for predicting the magnitude (i.e., size) by which the US/UK exchange rate changes overtime. However, the results also suggest that the rolling window forecasting method performs better based on the correct sign prediction criterion. In other words, the rolling window forecasting method is more appropriate for predicting the changes in the sign of the US/UK exchange rate.

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Introduction

This research employs the R software in identifying the most suitable ARMA model for forecasting the growth rate of the exchange rate between the US dollar and a unit of the British pound (denoted as \$/£). Although the whole dataset available contains observations from January 1971 to February 2020, all observations for January 1971 are excluded. This is due to the fact that the growth rate of the \$/£ exchange rate is measured by taking the first difference of its logarithmic transformation and this leads to losing the growth rate for January 1971. Hence, it becomes imperative to exclude all available observations for January 1971 so as to ensure consistency in the time periods.

Additionally, the data is systematically split into two distinct periods identified as the in-sample period and the out of sample period. The in-sample period spans from February 1971 to December 2012 while the out-of-sample period runs from January 2013 to February 2020. This research employs the best model selected for the in-sample period to make forecasts for the out-of-sample period. More comprehensively, this process uses the in-sample period for pre-analysis, unit root/stationarity tests, model selection and model diagnostics. Thereafter, the out-of-sample period is used for forecasting.

Generally, the results obtained from the pre-analysis and unit root/stationarity tests suggest that $\log(\$/\pounds)$ is non-stationary while $\Delta \log(\$/\pounds)$ is stationary. Also, the model selected by AIC is ARMA (2,3). Similarly, the model diagnostics do not show evidence of serial correlation in the residuals obtained from the regression estimated with the ARMA (2,3) model; and this suggests that the model fits the data well. As regards the plot of the true values against the predicted values, the forecasts generally provide a close fit for the true values, nonetheless, there exists a number of swings which are not forecasted.

Upon completing the above analysis, this research proceeds to make rolling window forecasts for the out-of-sample period. The predicted values obtained from the rolling window forecasting method are plotted against the true values of the out-of-sample period. The rolling window forecasts largely provide a close fit for the true values. Also, the forecasts appear to provide good predictions of

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the changes in the sign of the true values. Additionally, this research uses the MSE, MAE, MAPE and correct sign prediction criterion to compare the forecasting performance of the rolling window forecasting method with the traditional forecasting method. The results obtained suggest that the traditional forecasting method seems to perform better judging by MSE, MAE and MAPE. In other words, the traditional forecasting method appears to perform better in predicting the magnitude (i.e. size) by which $\Delta \log (\$/\pounds)$ series changes overtime. However, the results also suggest that the rolling window forecasting method seems to perform better based on the correct sign prediction criterion. In other words, the rolling window forecasting method appears to perform better in predicting the changes in the sign of $\Delta \log (\$/\pounds)$ series.

The rest of this research is organized as follows. Section 2 reviews the literature. Section 3 provides the methodology adopted. Section 4 discusses the results. Section 5 Concludes.

Literature Review

Comparative studies

Alam (2012) investigated the application of an AR model by comparison with the naïve strategy model for predicting the BDT/USD exchange rates over the period July 03, 2006 to July, 04, 2011 by dividing the data into in-sample and out of sample data set. The study finds that both the ARMA and AR models jointly outperform other models in predicting the BDT/USD exchange rate based on statistical forecasting measures.

Rout, Majhi, Majhi and Panda (2014) utilized both the adaptive autoregressive moving average (ARMA) technique and differential evolution (DE) methods. The Differential Evolution (DE) technique involves the imputation of past time series data into a prediction model to obtain coefficients. The prediction accuracy is validated using past exchange rates not previously included in the training procedure. Four alternatives methods namely ARMA-CSO, ARMA-BFO and ARMA:-FBLMS. A Comparisons of the different measures including the training time of the all three hybrid models provides evidence that the ARMA-DE exchange rate prediction model possesses superior short- and long-range prediction potentiality compared to others.

Using exchange rate data covering the period January 01, 2010, to April 30, 2015, and a total of 1284 observations. Babu and Reddy (2015) examined the predictability of exchange rates of Rupee against USD, GBP, Euro and Yen. Babu et al (2015) compared classical time series method (ARIMA) and complex nonlinear methods such as Neural Network and Fuzzy neurons. Daily RBI reference exchange rates from January 2010-April 2015 were used for the analysis in predicting exchange rate market in India ARIMA model does better than those of the complex nonlinear models.

Optimal ARMA Models (p,d,q)

Qonita, Pertiwi, and Widiyaningtyas (2017) investigated the best ARIMA method in predicting the value of the rupiah against the US dollar, using 1595 records on value, trading volume as well as the date, over the 30-day period between January 4, 2010 and June 24, 2016. Qonita et al (2017) followed four stages, including (1) using the ACF and partial ACF to determine the AR, the lag at which the model is integrated and the appropriate Moving Average cut off point (2) estimating the model using Maximum Likelihood to estimate the parameters (3) Diagnostic tests to detect white noise (4) Computing the MAE, MAPE and other statistical criteria of accuracy. The findings of Qonita et al (2017) shows that the most predictive model was the ARMA model (2,2) which had a predictive accuracy of about 98.74%.

Following a similar procedure, Nyoni (2019) adopted the Box-Jenkins ARIMA technique while utilizing data on the Indian Rupee to USD exchange rate from 1960 to 2017 as in-sample data in order to forecast exchange rate from 2018 – 2027. In this process, Nyoni (2019) found the ARIMA (0, 1, 6) technique sufficiently stable and appropriate for forecasting the Indian Rupee / USD exchange rates.

Ashour and Al-dahhan (2020) employed the Box-Jenkins methodology to develop a prediction model for the Turkish lira- the US dollar within the period 1/2/2018 - 30/6/2018. The RMSE and MAPE were used to measure the predictive accuracy. The results showed that the most reliable forecasting model was found to be ARMA (1,0).

Research and Methodology

Pre-analysis

Time series plot

A time series plot provides a graphical depiction of a variable whose data has been assembled over a given time-frame. While the horizontal axis measures time, the vertical axis measures the relevant variable. In time series plots, each data point is collected over the same intervals. While some time series plots may provide a graphical representation of a quantitative dataset (e.g. outstanding shares and exchange rates) others provide a graphical representation of a qualitative dataset (e.g. credit rating and financial products bought) (Brooks, 2014).

If the series of a variable is **stationary**, its time series plot would neither reflect a random walk (with or without a drift) nor show a trending behavior, also, it would frequently cross its mean value (Brooks, 2014).

Autocorrelation Function (ACF) plot

An ACF plot provides a graphical depiction of the correlation between a time series and various lags of itself. If the series of a variable is **stationary**, its autocorrelation with various lags of itself will hover around zero geometrically (Enders, 2015).

Given a time-series X_t , its ACF at lag k is expressed as: $\rho_k = \gamma_k / \gamma_0$

Where: γ_k = autocovariance of series X_t at lag k

$$\gamma_0 = \text{Variance of series } X_t$$

Box plot

A boxplot is a statistical technique which provides a standard depiction of a set of observations using five numbers. These five numbers are the minimum non-outlier, the first quartile, the median, the third quartile and the maximum non-outlier. The vertical line within a boxplot signifies the position of the median. Also, two whiskers extend from the left and right sides of the box. While the former represents the range of values between the first quartile and the smallest non-outlier, the latter represents the range of values between the third quartile and the largest non-outlier. Outliers are plotted as separate points on the left and right sides of the whiskers. As such, a unique property of the boxplot is that it provides an idea about the skewness and outliers within a dataset (see McGill, et al. 1978 and Mullenex, 1990)

If the series of a variable is **stationary** and normally distributed, the thick vertical line within the box will cut the box along the center and this suggests the boxplot is symmetric and normally distributed.

Descriptive statistics

Five major descriptive statistics are considered. They are: mean, median, standard deviation, skewness and kurtosis. Spiegel and Stephens (2014) define these statistics as follows:

Unit root/stationarity tests

Augmented Dickey Fuller (ADF) test

As a series with a unit root is non-stationary, a common method of testing the stationarity of a series is the unit root test and the ADF is a popular unit root test. The null hypothesis of the ADF test is that the relevant process has a unit root, while the alternative hypothesis is that it does not have a unit root. Before providing further technical description of the ADF test, it is necessary to describe the Dickey Fuller (DF) test. Given the AR(1) model:

$$X_t = \phi X_{t-1} + \varepsilon_t$$

If series X_t has a unit root, the estimated value of ϕ would be statistically equal to 1 and X_t would be considered a random walk without drift. The t-test cannot be used to confirm whether or not ϕ is equal to one, first, because it is used to check whether a parameter estimate is different from zero, and not one. Second, because it would be bias in the case of a unit root test. To solve this problem, Dickey and Fuller (1979) introduce an intuitive approach. They modify the original equation by subtracting X_{t-1} from both sides of the equation, such that we have:

$$X_t - X_{t-1} = \phi X_{t-1} - X_{t-1} + \varepsilon_t$$

$$\Delta X_t = (\phi - 1)X_{t-1} + \varepsilon_t$$

Replacing $\phi - 1$ with β , we have:

$$\Delta X_t = \beta X_{t-1} + \varepsilon_t$$

Mathematically, if β is statistically equal to zero, ϕ would be statistically equal to one, and the process in such a circumstance would have a unit root and thus be non-stationary. Similarly, if β is statistically less than zero, ϕ would be statistically less than one, and the process in such a circumstance would not be deemed to have a unit root and thus would be stationary. We can test if β is equal to zero with an approach similar to the t-test.

The DF test statistic is expressed as:

$$\text{DF test statistic} = \frac{\hat{\beta}}{SE(\hat{\beta})}$$

$SE(\hat{\beta})$ represents the standard error of $\hat{\beta}$. Although the DF test statistic may appear similar to the t-test statistic, they are not the same. While the t-test statistic follows a t-distribution, the DF statistic follows a non-standard normal distribution (with its critical values being obtained from Monte Carlo experiments). It is also important to observe that the DF statistic does not follow a t-distribution in

testing if $\hat{\beta}$ is equal to zero because X_{t-1} – which $\hat{\beta}$ is the coefficient of – may have a unit root, yet the results provided by a t-distribution will be highly biased in the presence of a unit root.

The decision rule with regards to the DF test statistic is that, if the t-statistic is greater than the test critical value, we reject the null hypothesis that $\hat{\beta}$ is equal to zero and as such the relevant series has no unit root. In this case, the p-value of the test statistic would be less than the chosen significance level (e.g., 0.05). However, if the t-statistic is less than the test critical value, we do not reject the null hypothesis that $\hat{\beta}$ is equal to zero and as such the relevant series has a unit root. In this case, the p-value of the test statistic would be greater than the chosen significance level.

The DF test can be conducted in three ways, which are highlighted below:

(I) Without allowing for a drift and a trend

$$\Delta X_t = \beta X_{t-1} + \varepsilon_t$$

(II) Allowing for a drift without allowing for a trend

$$\Delta X_t = \alpha + \beta X_{t-1} + \varepsilon_t$$

Where α represents the drift term

(III) Allowing for a drift and a trend

$$\Delta X_t = \alpha + \beta X_{t-1} + \lambda t + \varepsilon_t$$

Where t represents the trend term and λ is its corresponding coefficient.

Moving on to the ADF test; the DF test is constructed with the assumption that the error term is white noise, and this implies zero autocorrelation between the error term and various lags of itself. Yet if the dependent variable ΔX_t is autocorrelated with lags of itself, the error term may also be autocorrelated with lags of itself. To address this problem, Dickey and Fuller (1981) augment the DF test by introducing – among the explanatory variable – lags of the dependent variable ΔX_t in order to address potential autocorrelation issues within the error term ε_t . This improvement in the DF test is known as the Augmented Dickey Fuller test. The model used by the ADF is stated as follows:

$$\Delta X_t = \beta X_{t-1} + \sum_{i=1}^p \phi_i \Delta X_{t-i} + \varepsilon_t$$

The number of lags in the process range from i (which is equal to one) to p .

Dickey and Fuller (1981) identify three extra F-statistics referred to as ϕ_1 , ϕ_2 and ϕ_3 . As explained by Enders (2015), ϕ_1 is used in testing the joint hypothesis that the relevant series has a unit root and a drift. Also, ϕ_2 is used in testing the joint hypothesis that the relevant series has a unit root, a drift and a deterministic trend. Similarly, ϕ_3 is used in testing the joint hypothesis that the relevant series has a unit root and a deterministic trend.

Hence, the null hypothesis for ϕ_1 is that the relevant series has a unit root and a drift. Thus, the alternative hypothesis for ϕ_1 is that the relevant series does not have a unit root and/or a drift.

Similarly, the null hypothesis for ϕ_2 is that the relevant series has a unit root, a drift and a deterministic trend. Thus, the alternative hypothesis for ϕ_2 is that the relevant series does not have a unit root, and/or drift and/or a deterministic trend.

Likewise, the null hypothesis for ϕ_3 is that the relevant series has a unit root and a deterministic trend. Thus, the alternative hypothesis for ϕ_3 is that the relevant series does not have a unit root and/or a deterministic trend.

Phillips-Perron (PP) test

The Phillips-Perron (PP) test was introduced by Phillips and Perron (1988). Similar to the ADF test, the null hypothesis of the PP test is that the relevant process has a unit root, while the alternative hypothesis is that it does not have a unit root. Also, the PP test is based on the below model:

$$X_t = \phi X_{t-1} + \varepsilon_t$$

Nonetheless, the PP test corrects the ADF test in a manner such that the absence of serial correlation in the residuals is no longer a condition for conducting the unit root test. As such, the residuals can simply be stationary with a mean of zero.

As noted by Phillips (1987), the modified PP test statistics in the absence of deterministic components is expressed as:

$$Z - \alpha: T(\hat{\phi} - 1) - \frac{1}{2}(\hat{J}_T - \hat{\gamma}_\varepsilon(0)) \left(\frac{1}{T^2} \sum_{t=1}^T X_t^2 - 1 \right)^{-1}$$

$$Z - \tau: \sqrt{\frac{\hat{\gamma}_\varepsilon(0)}{\hat{J}_T}} t_{\hat{\phi}} - \frac{1}{2} \left(\hat{J}_T - \hat{\gamma}_\varepsilon(0) \right) \left(\frac{\hat{J}_T}{T^2} \sum_{t=1}^T X_{t-1}^2 \right)^{-1/2}$$

In the above formula, $\hat{\gamma}_\varepsilon(0)$ represents the unconditional variance estimated for the error term. Similarly, $\hat{J}_T$ represents long run variance estimated for the error term.

KPSS test

The KPSS test was introduced by Kwiatkowski et al. (1992). While the DF and PP tests are unit root tests. The KPSS test is a stationarity test. More specifically, the KPSS test swaps the null and alternative hypothesis in the ADF and PP tests. Hence, in the KPSS test, the null hypothesis is that the relevant series is stationary while the alternative hypothesis is that it is not stationary.

$$X_t = \alpha + \delta t + d \sum_{j=1}^t Z_j + U_t$$

Where α represents the drift term, t represents the deterministic trend, Z_j represents a set of white noise processes ranging from Z_1 to Z_t and U_t represents a stationary process. If d is equal to zero, the process would be considered trend-stationary. However, if d is not equal to zero, the process would be integrated of order one and hence non-stationary. Accordingly, the null hypothesis is that d is equal to zero while the alternative hypothesis is that d is not equal to zero.

Model selection

ARMA model

As discussed in Wang (2020), an ARMA model is one that combines an Autoregressive (AR) model and a Moving Average (MA) model. An AR model is a model which has as its explanatory variable(s), one or more lag(s) of the dependent variable. A simple AR process is stated below:

$$X_t = \phi X_{t-1} + \varepsilon_t$$

The above equation is known as a first order autoregressive process or an AR(1) process, as X_t depends on one period lag of itself. Also, ϕ is referred to as an autoregressive parameter. As X_t may depend on more than one lag of itself, it is a convention to denote the number of lags which X_t depends on as p . More technically, the value of p is known as the order of the AR process. An AR(p) process is stated below:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + \varepsilon_t$$

where $\phi_1, \phi_2, \dots, \phi_p$ represent the autoregressive parameters for each of the lags

On the other hand, an MA model is a model which has as its explanatory variables, the current white noise error term and one or more lags of this white noise error term. A simple MA process is stated below:

$$X_t = \varepsilon_t + \theta \varepsilon_{t-1}$$

The above equation is known as a first order moving average process or an MA(1) process as just one lag of the white noise error term is included in the model. Also, θ is referred to as the moving average parameter. As X_t may depend on more than one lag of the current white noise error term, it is a convention to denote the number of lags which X_t depends on as q . More technically, the value of q is known as the order of the MA process. An MA(q) process is stated below:

$$X_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_q \varepsilon_{t-q}$$

where $\theta_1, \theta_2, \dots, \theta_q$ represent the moving average parameters for each of the lags.

Hence, more generally, an MA process is a moving average of the current residuals as well as the previous ones.

Since an ARMA model combine an AR model as well as an MA model, it thus has as its explanatory variables: one or more lag(s) of the dependent variable, the current white noise error term and one or more lags of the current white noise error term.

A simple ARMA process is stated below:

$$X_t = \phi X_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$$

The above process is known as an ARMA(1, 1) process since X_t depends on just one period lag of itself and just one lag of the white noise error term is included in the model. As X_t may depend on more than one lag of itself and also more than one lag of the white noise error term, it is a convention to denote the ARMA process as ARMA(p, q). An ARMA (p, q) process is stated below:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_q \varepsilon_{t-q}$$

Selecting the best ARMA Model Using AIC

The AIC is popularly used in determining the order of the ARMA process. The AIC generates a single number to summarize the quality of various ARMA models of different orders. As it is based on an information criterion, the AIC generated for each ARMA model estimates the amount of information that would be lost to estimate the relevant series. Accordingly, the model with the least AIC is most preferred.

Model diagnostics

Time series plot of the residuals

The time series plot of the residuals provides a graphical depiction of the series of the error term obtained from the regression analysis conducted using the model selected by AIC. While the horizontal axis measures time, the vertical axis measures the error term. If the error term is **white noise**, its time series plot would neither reflect a random walk (with or without a drift) nor show a trending behavior, instead it would be similar to the classical pattern of a stationary process (Brooks, 2014).

Autocorrelation Function (ACF) plot of the residuals

An ACF plot of the residuals provides a graphical depiction of the correlation between the best ARMA model's error term and various lags of itself. If the series of the error term is not affected by serial correlation, its autocorrelation with various lags of itself will hover around zero geometrically (Enders, 2015).

Ljung-Box test on the residuals

The Ljung box Q statistics examines if the residuals obtained from an ARMA model follow the classical pattern of a white noise process. While the null hypothesis of this statistic is that the residuals are distributed independently (i.e. they are not autocorrelated), the alternative hypothesis is that they are not distributed independently. The null hypothesis is rejected if the p-value of the Ljung box Q statistic is less than the chosen significance level. Otherwise, it is not rejected.

The Ljung box Q statistic is stated below

$$\text{Ljung box Q statistics} = T(T+2) \sum_{h=1}^N \frac{\hat{\rho}_e^2(h)}{T-h} \sim \chi_N^2$$

where T represents the sample size, $\hat{\rho}$ represents the autocorrelation coefficient, h represents the number of lags. In practice, h is arbitrarily selected. Also, N represents the lag length. The Ljung box Q statistic follows the chi square distribution with N degrees of freedom (see Shumway and Stoffer, 2011).

Forecasting

Forecasting for out-of-sample period using model selected for in-sample period

In building a model robust enough to predict the future movements of a series, the dataset of the series is split into in-sample and out-of-sample period, such that the robust model is built using the data available for the in-sample period, and in order to check the forecasting power of this model, it is used in making forecasts for the out-of-sample period.

Plot of true values against predicted values

The plot of the true values of a series and its predicted values simply depicts the actual values of the series for the out-of-sample period and its forecasted values in a single graph. While the horizontal axis represents time, the vertical axis measures the forecasted and predicted values for the out-of-sample period of the series. The advantage of this plot is that it makes it possible to visually assess the forecasting power of the model built using the in-sample period data.

Estimation of model performance

As eye-balling the plot of the true and predicted values of a series only provides a rough idea about the forecasting power of the model built, more formal tests are necessary. These tests are discussed below:

Mean Squared Error (MSE)

The MSE computes the average of the squared forecasting errors for the entire out-of-sample period. As MSE works with the squared forecasting errors, one of its unique properties is that the treatment it gives to predictions that are above the actual value is similar to

the treatment it gives to predictions that are below the actual value. (Neusser 2016). The lower the MSE of a model, the greater is its forecasting performance. Hence, given two models, the one with the lower MSE is preferred.

MSE is calculated with the below formula:

$$MSE = \frac{1}{N} \sum_{t=1}^N (x_{t+s} - f_{t,s})^2$$

where N , x_{t+s} and $f_{t,s}$ represent the sample size, the true values and the forecasted values respectively.

Mean Absolute Error (MAE)

The MAE computes the average of the absolute forecast errors for the entire out-of-sample period. The lower the MAE of a model, the greater is its forecasting performance. Hence, given two models, the one with the lower MAE is preferred. MAE is calculated with the below formula:

$$MAE = \frac{1}{N} \sum_{t=1}^N |x_{t+s} - f_{t,s}|$$

Mean Absolute Percentage Error (MAPE)

MAPE is computed through three major steps. First, it takes the absolute value of the ratio of the forecast errors to the true values for each out-of-sample period. Second, it takes the average of this ratio. Third, this average is expressed as a percentage. As such, MAPE is unique in that it assesses the forecasting power of a model in percentage terms. In fact, MAPE arguably possesses the most desired properties expected to be found in a measure of model performance (Makridakis, 1993). Accordingly, MAPE is one of the most widely used measures of model performance. The lower the MAPE of a model, the greater is its forecasting performance. Hence, given two models, the one with the lower MAPE is preferred.

MAPE is calculated with the below formula:

$$MAPE = \frac{100}{N} \sum_{t=1}^N \left| \frac{x_{t+s} - f_{t,s}}{x_{t+s}} \right|$$

Correct sign prediction criterion

The correct sign prediction criterion is used in situations wherein the objective behind making forecasts is solely to predict the changes in the sign of a series, whether positive or negative. Accordingly, the correct sign prediction criterion is very useful for forecasting whether the returns on a stock would be positive or negative. When investing on a stock, investors are particular about positive returns even if they are little.

As discussed by Brooks (2014), the correct sign prediction criterion is calculated with the below formula:

$$\text{Correct Sign Prediction Criterion} = \frac{1}{N} \sum_{t=1}^N z_{t+s}$$

Where $z_{t+s} = 1$ if $(X_{t+s} \cdot f_{t,s}) > 0$

$z_{t+s} = 0$ otherwise

In the above formula, z_{t+s} takes the value of 1 if in a given period, the product of the actual value and the forecasted value is positive. However, if this does not hold, z_{t+s} takes the value of zero. As the product of the actual and forecasted values will be positive only when both have the same signs (i.e. either both are positive or both are negative), it can be said that z_{t+s} takes the value of 1 whenever the model correctly forecasts the sign of the actual values, while it takes the value of 0, whenever the model wrongly predicts the sign of the actual values.

It can thus be inferred that the correct sign prediction provides information about the proportion of signs a model predicts correctly. Given two models, the one with the higher correct sign prediction is preferred if the objective behind making forecasts is solely to predict the changes in the sign of a series.

Rolling-window Forecasting

In the rolling window forecast, forecasts are iteratively made for just one-step ahead. Specifically, the rolling window forecast proceeds by first using the best model selected for the initial in-sample period to make a forecast for just one step ahead, thereafter, another one-step ahead forecast is made with the best model for a new sample created by excluding the first observation from the in-

sample period and including a new observation at its end. Thereafter, another one-step ahead forecast is made similarly and this process is repeated until a final one-step ahead forecast is made for the last observation available. (Brooks, 2014).

As such, there are two major differences between the traditional forecasting method and the rolling window forecast. First, in the traditional forecasting method, the in-sample period is used to make forecasts for multiple periods ahead, however, in the rolling window forecast, the in-sample period is used to make one-step ahead forecast. Second, in the traditional forecasting method, absolutely no changes are made to the in-sample period data, however in rolling window forecasts, successive one-step ahead forecasts are made by excluding the first observation from the in-sample period and including a new observation at its end.

It is necessary to note that the rolling window forecasting technique is relevant in a situation in which the data generating process of a series evolves over-time. In such a situation, the traditional forecasting method may be less accurate. The reason for this is that as forecasts are made for each successive period, the old observations in the initial in-sample period would become less representative of late periods, similarly, the recent observations in it would become less representative of early periods. Accordingly, in this kind of situation, it would be useful to methodically drop the oldest observations and include most recent observations, and this is exactly what is done in the rolling window forecasting method.

Results

Pre-analysis

Discussion of results on $\log (\$/\pounds)$

Time series plot of $\log (\$/\pounds)$

Figure 1. shows the time series plot of $\log (\$/\pounds)$. While the horizontal axis measures the timeframe under consideration, the vertical axis measures the corresponding values of $\log (\$/\pounds)$. Although the time-series plot of $\log (\$/\pounds)$ does not seem to show any trending or explosive behaviour, it however appears to meander in a manner similar to that of a random walk process. As such, the plot does not seem to depict a stationary time series.

Figure 1: Time series plot of $\log (\$/\pounds)$

Autocorrelation Function (ACF) plot of $\log (\$/\pounds)$

Figure 2. depicts the ACF plot of $\log (\$/\pounds)$. The ACF plot clearly shows that the autocorrelations decay geometrically and do not hover around zero and this runs contrary to the known pattern of stationary processes.

Figure 2: ACF plot of $\log (\$/\pounds)$

Box plot of $\log (\$/\pounds)$

Figure 3. shows the box plot of $\log (\$/\pounds)$ over the timeframe under consideration. Although the precise values of the five-number summary are not clear from the box plot, it can be deduced that the minimum non-outlier is less than 0.2. Both the first quartile and the median are greater than 0.4 but less than 0.6. However, while the first quartile is closer to 0.4, the median is closer to 0.6. Also,

the third quartile is greater than 0.6 but less than 0.8; nonetheless, it is closer to 0.6. Also, the maximum non-outlier is greater than 0.8.

Figure 3: Box plot of $\log (\$/\pounds)$

Similarly, the whiskers of the box plot suggest that there exist some outliers that are lower than the minimum non-outlier. However, no outliers are reported to be greater than the maximum non-outlier. Ordinarily, if a series is normally distributed, the vertical line within the box will cut the box along the center, however, in this case, the vertical line does not cut the box along the center, but instead cuts the box in a manner such that the right section of the box is wider than the left one and this indicates the series is not symmetric but skewed to the right.

Descriptive statistics of $\log (\$/\pounds)$

Table 1 shows the descriptive statistics for $\log (\$/\pounds)$. The descriptive statistics reported are the mean, median, standard deviation, skewness and kurtosis. While the mean is 0.557, the median is 0.512 and the standard deviation is 0.168. The skewness of 0.541 is positive and this confirms the frequency distribution of the series is skewed to the right (i.e. has a longer tail to the right of the central maximum). This suggests that the frequency distribution of the series may be different from what obtains in a symmetrical or normal distribution. Also, the kurtosis of 2.8 which is less than 3 indicates that the frequency distribution may be platykurtic and hence may have fewer extreme observations than a normal distribution.

Table 1: Descriptive statistics for $\log (\$/\pounds)$ and $\Delta \log (\$/\pounds)$

	$\log (\$/\pounds)$	$\Delta \log (\$/\pounds)$
Mean	0.55750	-0.00079
Median	0.51246	-0.00012
Sd	0.16830	0.02407
Skewness	0.54171	-0.26237
Kurtosis	2.86487	4.69913

Source: Author's computation

Discussion of results on $\Delta \log (\$/\pounds)$

Time series plot of $\Delta \log (\$/\pounds)$

Figure 5 shows the time series plot of $\Delta \log (\$/\pounds)$. While the horizontal axis measures the timeframe under consideration, the vertical axis measures the corresponding values of $\Delta \log (\$/\pounds)$. The time-series plot of $\Delta \log (\$/\pounds)$ neither shows a trending behaviour nor appears to meander in a manner similar to that of a random walk process. As such, the plot seems to depict a stationary time series.

Figure 5: CF plot of $\Delta \log (\$/\pounds)$

Autocorrelation Function (ACF) plot of $\Delta \log (\$/\pounds)$

Figure 6. depicts the ACF plot of $\Delta \log (\$/\pounds)$. The ACF plot clearly shows that the autocorrelations converge to zero after one lag. After this lag, the ACF hovers around zero, and this is very much in line with the known pattern of stationary processes.

Figure 6: Box plot of $\Delta \log (\$/\pounds)$

Box plot of $\Delta \log (\$/\pounds)$

Figure 7 shows the box plot of $\Delta \log (\$/\pounds)$ over the timeframe under consideration. Although the precise values of the five-number summary is not clear from the box plot, it can be inferred that the minimum non-outlier is greater than -0.10 but less than -0.05. The first quartile is greater than -0.05 but less than 0.00. While the median lies close to 0.00, the third quartile lies between 0.00 and 0.05, although it is closer to 0.00. Also, the maximum non-outlier is slightly greater than 0.05.

Figure 7: Time series plot of the residuals of the model selected by AIC

Similarly, the whiskers of the box plot suggest that there exist some outliers that are lower than the minimum non-outlier as well as those that are greater than the maximum non-outlier. The thick vertical line within the box seems to cut the box along the center, and this may suggest that series $\Delta \log (\$/\pounds)$ is closer to being normally distributed compared to series $\log (\$/\pounds)$. Nonetheless, it may be inadequate to rely solely on the box plot to draw conclusions about the existence of normality, as the boxplot only provides an idea about the distribution. Hence, more rigorous statistical techniques are considered below.

Descriptive statistics of $\Delta \log (\$/\pounds)$

Table 1 (see above table 1 above). shows the descriptive statistics for $\Delta \log (\$/\pounds)$. While the mean is -0.00079, the median is -0.00012 and the standard deviation is 0.0240.

The skewness of -0.26237 is close to zero but slightly negative and this may suggest that the frequency distribution of the series is slightly skewed to the left (i.e. has a longer tail to the left of the central maximum). This suggests that the frequency distribution of the series may be different from what obtains in a symmetrical or normal distribution. Also, the kurtosis of 4.69913 is greater than 3 and this also indicates that the frequency distribution may be leptokurtic and may also have more extreme observations than a normal distribution.

Unit root/Stationarity tests

Unit root/ stationarity tests on $\log (\$/\pounds)$

General points concerning how the setting is chosen

Table 2 shows the results obtained from ADF, PP and KPSS stationarity tests of series $\log (\$/\pounds)$. It is necessary to state that the regression estimated in each of these stationarity tests allows for a drift but not a trend. This has been done for the following reasons:

First, a drift is included because the time series plot of $\log (\$/\pounds)$ appears to meander in a manner similar to that of a random walk process. This is further underscored by the fact that the ACF plot of $\log (\$/\pounds)$ shows that the autocorrelations decay geometrically and do not hover around zero.

The absence of a deterministic trend is further confirmed by the fact that the $\log (\$/\pounds)$ series declines significantly all through the early 1980s and increases significantly all through the mid-1980s. For this reason, a deterministic trend is not accounted for in the unit root/stationarity tests.

Table 2: Unit root/stationarity tests on $\log(\$/\pounds)$

Test	H0	Stat	CV	Reject?	Conclusion
ADF	$\log(\$/\pounds)$ has a unit root	-2.928	-2.867	reject	stationary
PP	$\log(\$/\pounds)$ has a unit root	-2.526	-2.867	not reject	non-stationary
KPSS	$\log(\$/\pounds)$ is stationary	2.556	0.463	reject	non-stationary

Source: Author's Computation

ADF unit root test on $\log(\$/\pounds)$

In conducting the ADF test, the AIC is set at a maximum of 20 lags, however, the AIC selected just three lags of the dependent variable.

As seen in Table 2 the test statistic for ADF test is -2.928 while the test critical value at 5% significance level is -2.867. Since the absolute value of the test statistic is greater than the absolute value of the test critical value, we reject the null hypothesis that $\log(\$/\pounds)$ has a unit root. Nonetheless, as seen in Appendix A, ϕ_1 statistic is 4.410 while the corresponding critical value at the 5% significance level is 4.590. Since, the ϕ_1 statistic is less than its critical value, we do not reject the joint hypothesis that $\log(\$/\pounds)$ has a unit root and a drift. As such, the results obtained from the ADF test are inconclusive thereby necessitating the need for further unit root/stationarity tests.

PP unit root test on $\log(\$/\pounds)$

In conducting the PP test, the default number of lags specified by the Newey-West estimator is used. As such the "short" lag length is used on R software.

As seen in Table 2 the test statistic for the PP test is -2.526 while the test critical value at the 5% significance level is -2.867. Since the absolute value of the test statistic is less than the absolute value of the test critical value, we do not reject the null hypothesis that $\log(\$/\pounds)$ has a unit root.

KPSS stationarity test on $\log(\$/\pounds)$

Also, in conducting the KPSS test, the default number of lags specified by the Newey-West estimator is used. As seen in Table 2 the test statistic for the KPSS test is 2.556 while the test critical value at the 5% significance level is 0.463. Since the test statistic is greater than the test critical value, we reject the null hypothesis that $\log(\$/\pounds)$ is stationary.

Unit root/ stationarity tests on $\Delta \log (\$/\pounds)$

General points concerning how the setting is chosen

Table 3 shows the results obtained from ADF, PP and KPSS stationarity tests of series $\Delta \log (\$/\pounds)$. Again the regression estimated in each of these stationarity tests allows for a drift but not a trend. This has been done for the following reasons:

First, a drift is included since the mean of the series is still non-zero, although quite close to zero.

Second, a trend is not included because no trending behaviour is observed in the time series plot of $\Delta \log (\$/\pounds)$.

Table 3: Unit root/stationarity tests on $\Delta \log (\$/\pounds)$

Test	H0	Stat	CV	Reject?	Conclusion
ADF	$\Delta \log (\$/\pounds)$ has a unit root	-10.624	-2.867	reject	stationary
PP	$\Delta \log (\$/\pounds)$ has a unit root	-15.317	-2.867	reject	stationary
KPSS	$\Delta \log (\$/\pounds)$ is stationary	0.0745	0.463	not reject	stationary

Source: Author's Computation

ADF unit root test on $\Delta \log (\$/\pounds)$

In conducting the ADF test, the AIC is set at a maximum of 20 lags, however, the AIC selected just two lags of the dependent variable.

As seen in Table 3 the test statistic for ADF test is -10.624 while the test critical value at 5% significance level is -2.867. Since the absolute value of the test statistic is greater than the absolute value of the test critical value, we reject the null hypothesis that $\Delta\log(\$/\pounds)$ has a unit root. Also, as seen in Appendix A, ϕ_1 statistic is 56.444 while the corresponding critical value at the 5% significance level is 4.590. Since, the ϕ_1 statistic is greater than its critical value, we conclude that $\Delta\log(\$/\pounds)$ does not have a unit root and/or a drift.

PP unit root test on $\Delta\log(\$/\pounds)$

In conducting the PP test, the default number of lags specified by the Newey-West estimator is used.

As seen in Table 3. the test statistic for the PP test is -15.317 while the test critical value at the 5% significance level is -2.867. Since the absolute value of the test statistic is greater than the absolute value of the test critical value, we reject the null hypothesis that $\Delta\log(\$/\pounds)$ has a unit root.

KPSS stationarity test on $\Delta\log(\$/\pounds)$

Also, in conducting the KPSS test, the default number of lags specified by the Newey-West estimator is used.

As seen in Table 3 the test statistic for the KPSS test is 0.0745 while the test critical value at the 5% significance level is 0.463. Since the test statistic is less than the test critical value, we do not reject the null hypothesis that $\Delta\log(\$/\pounds)$ is stationary.

Model Selection

In using the AIC to select the best model, the maximum lag length is set to 5. Table 4 shows that the AIC of ARMA (2,3) model is -2396.018 and this represents the minimum AIC among the all possible 36 ARMA models. As such, the ARMA (2,3) model is selected as the best model by AIC.

Table 4: AIC of all ARMA (p,q) models

p\q	0	1	2	3	4	5
0	-2318.465	-2393.039	-2391.062	-2389.305	-2390.133	-2388.489
1	-2383.710	-2391.065	-2389.364	-2387.910	-2388.449	-2387.255
2	-2387.626	-2389.225	-2387.717	-2396.018	-2386.454	-2392.051
3	-2390.826	-2388.951	-2387.300	-2385.300	-2392.479	-2390.857
4	-2389.061	-2388.236	-2385.302	-2386.607	-2390.970	-2394.418
5	-2388.128	-2385.976	-2385.038	-2387.456	-2392.090	-2392.885

Source: Author's Computation

Model Diagnostics

Time series plot of the residuals

The time series plot of the series of the error term obtained from the regression analysis conducted using the model selected by AIC (i.e. the ARMA (2,3) model) has been demonstrated. While the horizontal axis measures time, the vertical axis measures the error term. The time-series plot of the error term neither shows a trending behaviour nor appears to meander in a manner similar to that of a random walk process. The plot seems to depict a stationary time series.

Autocorrelation Function (ACF) plot of the residuals

The ACF plot of the error term obtained from the regression analysis conducted using the model selected by AIC has been demonstrated. The ACF plot clearly shows that the autocorrelations hover around zero, and this confirms that the series of the error term is not affected by serial correlation.

Ljung-Box test on the residuals

Table 5 shows the results obtained from the Ljung box test on the residuals. At lags 20, 30 and 50, the p-values for the Ljung box test are 0.500, 0.363 and 0.157 respectively. Since these p-values are greater than all the conventional significance levels (i.e. 0.01, 0.05 and 0.1), we can conclude that the lags are not significantly different from zero and hence, we do not reject the null hypothesis that the residuals are independently distributed at lags 20, 30 and 50. This also suggests overwhelmingly that the residuals are not autocorrelated and the ARMA (2,3) model fits the $\Delta\log(\$/\pounds)$ data decently.

Table 5: Ljung-Box autocorrelation test on the residuals of the model selected by AIC

	P-value
Lag 20	0.500
Lag 30	0.363
Lag 50	0.157

Source: Author's Computation

Forecasting

General point concerning forecasting for out-of-sample period using model selected for in-sample period

As mentioned in the introduction, the main data set used in this analysis for $\Delta \log(\$/\pounds)$ spans over the period February 1971–February 2020. However, the aim of this analysis is to build a model that is robust enough to predict the future movements of $\Delta \log(\$/\pounds)$. To achieve this, the dataset for $\Delta \log(\$/\pounds)$ is split into in-sample (February 1971–December 2012) and out-of-sample period (January 2013– February 2020), such that a robust model is built using the in-sample period data, and in order to check the forecasting power of this model, it is used in making forecasts for the out-of-sample period.

Forecasting results

Table 6 and Table 7 show the forecasting results for $\Delta \log(\$/\pounds)$. While the highest forecasting error is 0.031 (3.1%), the lowest forecasting error is -0.0778 (-7.778%). Both forecasting errors are recorded in September, 2017 and July, 2016 respectively. Also, the average of the forecasting error is -0.0018 (-0.18%).

Plot of true values of $\Delta \log(\$/\pounds)$ against predicted values

The plot of the true values of $\Delta \log(\$/\pounds)$ against its predicted values simply depicts the actual values of $\Delta \log(\$/\pounds)$ for the out-of-sample period and its forecasted values in a single graph. While the horizontal axis represents time, the vertical axis measures the forecasted and predicted values of $\Delta \log(\$/\pounds)$ for the out-of-sample period. Figure 9 shows the plot of the true values of $\Delta \log(\$/\pounds)$ against its predicted values. Although there exists a number of swings which are not forecasted, the forecasts generally provide a close fit for the true values.

Estimation of model performance

Table 8. shows the MSE, MAE, MAPE and correct sign prediction obtained upon forecasting with the ARMA(2,3) model. As seen in the table, the MSE, MAE, MAPE and the correct sign prediction values are 0.000381, 0.0149, 120.821 and 0.5 respectively. To draw robust inferences from these values, we can compare them with the model performance estimates obtained for another ARMA model. Hence, in the next section, we make use of the rolling window forecasting method and compare its forecasting performance with the one used in this section. Figure 8 shows ACP plot of the residuals of the model selected by AIC while Figure 9 shows the plot of the true values against the predicted values

Figure 8: ACF plot of the residuals of the model selected by AIC

Figure 9: Plot of the true values against the predicted values

Table 6: Forecasting Results

Time	True Value	Forecasted Value	Forecasting Error
Jan2013	-0.0112	0.0009	-0.0121
Feb2013	-0.0312	-0.0041	-0.0271
Mar2013	-0.0257	-0.0013	-0.0244
Apr2013	0.0152	0.0016	0.0136
May2013	-0.0009	0.0030	-0.0039
Jun2013	0.0127	0.0021	0.0106
Jul2013	-0.0204	-0.0005	-0.0199
Aug2013	0.0212	-0.0032	0.0244
Sep2013	0.0242	-0.0043	0.0285
Oct2013	0.0133	-0.0032	0.0165
Nov2013	0.0001	-0.0007	0.0008
Dec2013	0.0174	0.0015	0.0159
Jan2014	0.0052	0.0024	0.0028
Feb2014	0.0053	0.0012	0.0041
Mar2014	0.0039	-0.0010	0.0049
Apr2014	0.0074	-0.0031	0.0105
May2014	0.0055	-0.0037	0.0092
Jun2014	0.0039	-0.0025	0.0064
Jul2014	0.0093	-0.0003	0.0096
Aug2014	-0.0216	0.0014	-0.0230
Sep2014	-0.0248	0.0018	-0.0266
Oct2014	-0.0133	0.0006	-0.0139
Nov2014	-0.0109	-0.0013	-0.0177
Dec2014	-0.008	-0.0029	-0.0051
Jan2015	-0.0326	-0.0031	-0.0295
Feb2015	0.0122	-0.0019	0.0141
Mar2015	-0.0245	-0.0001	-0.0244
Apr2015	0.0006	0.0012	-0.0006
May2015	0.0320	0.0012	0.0308
Jun2015	0.0077	0.0001	0.0076
Jul2015	-0.0010	-0.0015	0.0005
Aug2015	0.0011	-0.0026	0.0037
Sep2015	-0.0155	-0.0026	-0.0129
Oct2015	0.0003	-0.0014	0.0017
Nov2015	-0.0097	0.0000	-0.0097
Dec2015	-0.0141	0.0009	-0.0150
Jan2016	-0.0401	0.0008	-0.0409
Feb2016	-0.0071	-0.0002	-0.0069
Mar2016	-0.0028	-0.0016	-0.0012
Apr2016	0.0049	-0.0024	0.0073
May2016	0.0142	-0.0022	0.0164
Jun2016	-0.0227	-0.0011	-0.0216
Jul2016	-0.0778	0.0000	-0.0778
Aug2016	-0.0025	0.0007	-0.0032
Sep2016	0.0029	0.0004	0.0025
Oct2016	-0.0636	-0.0005	-0.0631
Nov2016	0.0082	-0.0016	0.0098
Dec2016	0.0040	-0.0021	0.0061
Jan2017	-0.0093	-0.0018	-0.0075
Feb2017	0.0102	-0.0008	0.0110
Mar2017	-0.0119	0.0000	-0.0119
Apr2017	0.0233	0.0004	0.0229
May2017	0.0226	0.0001	0.0225
Jun2017	-0.0092	-0.0007	-0.0085
Jul2017	0.0144	-0.0016	0.0160
Aug2017	-0.0033	-0.0019	-0.0014
Sep2017	0.0295	-0.0015	0.0310
Oct2017	-0.0103	-0.0007	-0.0096
Nov2017	0.0011	0.0000	0.0011
Dec2017	0.0140	0.0002	0.0138
Jan2018	0.0308	-0.0001	0.0309
Feb2018	0.0098	-0.0009	0.0107
Mar2018	0.0010	-0.0015	0.0025
Apr2018	0.0073	-0.0017	0.0090
May2018	-0.0442	-0.0013	-0.0429
Jun2018	-0.0131	-0.0005	-0.0126
Jul2018	-0.0099	0.0000	-0.0099
Aug2018	-0.0218	0.0000	-0.0218
Sep2018	0.0144	-0.0003	0.0147
Oct2018	-0.0041	-0.0010	-0.0031
Nov2018	-0.0086	-0.0015	-0.0071
Dec2018	-0.0184	-0.0015	-0.0169
Jan2019	0.0185	-0.0011	0.0196
Feb2019	0.0088	-0.0005	0.0093
Mar2019	0.0115	-0.0001	0.0116
Apr2019	-0.0105	-0.0001	-0.0104
May2019	-0.0134	-0.0005	-0.0129
Jun2019	-0.0141	-0.0010	-0.0131
Jul2019	-0.0170	-0.0014	-0.0156
Aug2019	-0.0244	-0.0013	-0.0231
Sep2019	0.0170	-0.0009	0.0179
Oct2019	0.0230	-0.0004	0.0234
Nov2019	0.0177	-0.0002	0.0179
Dec2019	0.0173	-0.0002	0.0175
Jan2020	-0.0025	-0.0006	-0.0019
Feb2020	-0.0094	-0.0011	-0.0083

Source: Author's Computation

Rolling Window Forecasting

General point concerning the rolling window forecasting method used

The rolling window forecast proceeds by first using the ARMA (2,3) model selected for the in-sample period in the traditional forecasting method (i.e. February 1971 to December 2012) to make a forecast for January 2013 only, thereafter, a forecast is made for February 2013 only using the best model for a new sample created by excluding February 1971 from the in-sample period and including January 2013 at its end. Thereafter, a forecast is similarly made for March 2013 only, and this process is repeated until a final forecast is made for February 2020 only. Accordingly, the number of observations used to make each forecast remains the same (i.e. 503 observations).

Forecasting results

Table 9 shows the forecasting results for $\Delta \log(\$/\pounds)$. While the highest forecasting error is 0.0348 (3.48%), the lowest forecasting error is -0.0693 (-6.93%). Both forecasting errors are recorded in September 2017 and July 2016 respectively. Also, the average of the forecasting error is -0.00139 (-0.139%).

Model selection and diagnostics procedures

The best model selected by AIC in different cases were ARMA (2,3), ARMA(4,5), ARMA (0,1), ARMA (4,1) and ARMA (1,5). However, the best model selected in most cases is the ARMA (2,3) model. Also, the p-values for the Ljung box statistic on the residuals of all models selected are greater than the conventional significance levels, at lag 20, lag 30 and lag 50, thus confirming the residuals are not autocorrelated. The fact that the residuals are not autocorrelated is further confirmed by their time series and ACF plots (see Appendix B, Appendix C and Appendix D) – which are consistent with the classical patterns of a white noise process.

Table 7: Forecasting results expressed in percentage terms

Time	True Value	Forecasted Value	Forecasting Error
Jan2013	-1.1211%	0.0956%	-1.2167%
Feb2013	-3.1237%	-0.4187%	-2.7050%
Mar2013	-2.5791%	-0.1362%	-2.4429%
Apr2013	1.5202%	0.1671%	1.3531%
May2013	-0.0914%	0.3090%	-0.4004%
Jun2013	1.2731%	0.2122%	1.0609%
Jul2013	-2.0475%	-0.0544%	-1.9931%
Aug2013	2.1249%	-0.3236%	2.4485%
Sep2013	2.4212%	-0.4343%	2.8555%
Oct2013	1.3319%	-0.3276%	1.6595%
Nov2013	0.0124%	-0.0775%	0.0899%
Dec2013	1.7424%	0.1598%	1.5826%
Jan2014	0.5296%	0.2432%	0.2864%
Feb2014	0.5328%	0.1298%	0.4030%
Mar2014	0.3978%	-0.1033%	0.5011%
Apr2014	0.7431%	-0.3113%	1.0544%
May2014	0.5596%	-0.3710%	0.9306%
Jun2014	0.3911%	-0.2536%	0.6447%
Jul2014	0.9301%	-0.0374%	0.9675%
Aug2014	-2.1679%	0.1435%	-2.3114%
Sep2014	-2.4857%	0.1829%	-2.6686%
Oct2014	-1.3348%	0.0636%	-1.3984%
Nov2014	-1.903%	-0.1356%	-1.7674%
Dec2014	-0.8085%	-0.2920%	-0.5165%
Jan2015	-3.2615%	-0.3140%	-2.9475%
Feb2015	1.2274%	-0.1947%	1.4221%
Mar2015	-2.4500%	-0.0120%	-2.4380%
Apr2015	0.0668%	0.1219%	-0.0551%
May2015	3.2082%	0.1293%	3.0789%
Jun2015	0.7733%	0.0116%	0.7617%
Jul2015	-0.1027%	-0.1550%	0.0523%
Aug2015	0.1156%	-0.2687%	0.3843%
Sep2015	-1.5526%	-0.2639%	-1.2887%
Oct2015	0.0325%	-0.1490%	0.1815%
Nov2015	-0.9758%	0.0021%	-0.9779%
Dec2015	-1.4117%	0.0975%	-1.5092%
Jan2016	-4.0110%	0.0828%	-4.0938%
Feb2016	-0.7112%	-0.0282%	-0.6830%
Mar2016	-0.2873%	-0.1646%	-0.1227%
Apr2016	0.4900%	-0.2438%	0.7338%
May2016	1.4215%	-0.2209%	1.6424%
Jun2016	-2.2771%	-0.1145%	-2.1626%
Jul2016	-7.7826%	0.0079%	-7.7905%
Aug2016	-0.2515%	0.0725%	-0.3240%
Sep2016	0.2972%	0.0433%	0.2539%
Oct2016	-6.3625%	-0.0578%	-6.3047%
Nov2016	0.8238%	-0.1672%	0.9910%
Dec2016	0.4093%	-0.2190%	0.6283%
Jan2017	-0.9336%	-0.1849%	-0.7487%
Feb2017	1.0296%	-0.0892%	1.1188%
Mar2017	-1.1915%	0.0078%	-1.1993%
Apr2017	2.3374%	0.0482%	2.2892%
May2017	2.2685%	0.0105%	2.2580%
Jun2017	-0.9246%	-0.0791%	-0.8455%
Jul2017	1.4415%	-0.1648%	1.6063%
Aug2017	-0.3391%	-0.1954%	-0.1437%
Sep2017	2.9516%	-0.1552%	3.1068%
Oct2017	-1.0398%	-0.0715%	-0.9683%
Nov2017	0.1135%	0.0035%	0.1100%
Dec2017	1.4049%	0.0256%	1.3793%
Jan2018	3.0853%	-0.0161%	3.1014%
Feb2018	0.9861%	-0.0937%	1.0798%
Mar2018	0.1073%	-0.1592%	0.2665%
Apr2018	0.7342%	-0.1738%	0.9080%
May2018	-4.4219%	-0.1314%	-4.2905%
Jun2018	-1.3152%	-0.0597%	-1.2555%
Jul2018	-0.9978%	-0.0032%	-0.9946%
Aug2018	-2.1813%	0.0051%	-2.1864%
Sep2018	1.4493%	-0.0373%	1.4866%
Oct2018	-0.4141%	-0.1030%	-0.3111%
Nov2018	-0.8644%	-0.1515%	-0.7129%
Dec2018	-1.8463%	-0.1546%	-1.6917%
Jan2019	1.8541%	-0.1126%	1.9667%
Feb2019	0.8874%	-0.0526%	0.9400%
Mar2019	1.1534%	-0.0114%	1.1648%
Apr2019	-1.0536%	-0.0127%	-1.0409%
May2019	-1.3444%	-0.0538%	-1.2906%
Jun2019	-1.4101%	-0.1082%	-1.3019%
Jul2019	-1.7027%	-0.1429%	-1.5598%
Aug2019	-2.4451%	-0.1379%	-2.3072%
Sep2019	1.7041%	-0.0982%	1.8023%
Oct2019	2.3017%	-0.0490%	2.3507%
Nov2019	1.7775%	-0.0203%	1.7978%
Dec2019	1.7312%	-0.0282%	1.7594%
Jan2020	-0.2520%	-0.0663%	-0.1857%
Feb2020	-0.9451%	-0.1104%	-0.8347%

Source: Author's Calculation

Table 8: Forecasting performance through MSE, MAE, MAPE and correct sign prediction criterion

	Performance value
MSE	0.0003819963
MAE	0.01493627
MAPE	120.8214
Correct sign	0.5

Source: Author's Computation

Plot of predicted values against true values

Upon making rolling window forecasts for the periods spanning from January 2013–February 2020 (i.e. 86 periods), all the predicted values of $\Delta \log(\$/\pounds)$ are collected and Figure shows the plot of these predicted values against their true values.

The forecasts largely provide a close fit for the true values. Also, the forecasts appear to provide good predictions of the changes in the sign of the true values.

Evaluating model performance

Table 8 shows the MSE, MAE, MAPE and correct sign prediction obtained upon using the rolling window forecasting method. As seen in the table, the MSE, MAE, MAPE and the correct sign prediction values are 0.000405, 0.0159, 223.49 and 0.569 respectively.

Comparing the forecasting performance of the traditional forecasting method and the rolling window forecasting method

By comparing Table 8 and Table 10 it is observed that the MSE, MAE and MAPE of the traditional forecasting method is lower than that of the rolling window forecasting method and this suggests that, judging by MSE, MAE and MAPE, the traditional forecasting method should be preferred over the rolling window forecasting method. More specifically, these results indicate that the traditional forecasting method seems to perform better in predicting the magnitude (i.e. size) by which $\Delta \log(\$/\pounds)$ changes overtime.

However, upon comparing Table 8 and Table 10. it is observed that the correct sign prediction criterion of the rolling window forecasting method is higher than that of the traditional forecasting method and this suggests that, judging by the correct sign prediction criterion, the rolling window forecasting method should be preferred over the traditional forecasting method. More specifically, these results indicate that the rolling window forecasting method seems to perform better in predicting the changes in the sign of $\Delta \log(\$/\pounds)$ series. This is unsurprising as the time series plot of the rolling window forecasts appear to provide a better prediction of the ups and downs in the movement of $\Delta \log(\$/\pounds)$ series.

Figure 10: Plot of the true values against the values predicted by the rolling window forecasting method

Table 9: Rolling window forecast results expressed in percentage terms

Time	True Value	Forecasted Value	Forecasting Error
Jan2013	-1.1211%	0.0956%	-1.2167%
Feb2013	-3.1237%	-0.9014%	-2.2223%
Mar2013	-2.5791%	-0.6146%	-1.9645%
Apr2013	1.5202%	-0.5568%	2.077%
May2013	-0.0914%	0.9028%	-0.9942%
Jun2013	1.2731%	-0.373%	1.6461%
Jul2013	-2.0475%	0.5732%	-2.6207%
Aug2013	2.1249%	-1.1807%	3.3056%
Sep2013	2.4212%	1.0397%	1.3815%
Oct2013	1.3319%	0.3239%	1.008%
Nov2013	0.0124%	0.3825%	-0.3701%
Dec2013	1.7424%	0.0426%	1.6998%
Jan2014	0.5296%	0.8780%	-0.3484%
Feb2014	0.5328%	-0.0598%	0.5926%
Mar2014	0.3978%	0.0988%	0.2990%
Apr2014	0.7431%	-0.1748%	0.9179%
May2014	0.5596%	0.0603%	0.4993%
Jun2014	0.3911%	0.0357%	0.3554%
Jul2014	0.9301%	0.1715%	0.7586%
Aug2014	-2.1679%	0.4399%	-2.6078%
Sep2014	-2.4857%	-0.8964%	-1.5893%
Oct2014	-1.3348%	-0.6572%	-0.6776%
Nov2014	-1.903%	-0.5685%	-1.3345%
Dec2014	-0.8085%	-1.0142%	0.2057%
Jan2015	-3.2615%	-0.3880%	-2.8735%
Feb2015	1.2274%	-1.6093%	2.8367%
Mar2015	-2.4500%	1.0293%	-3.4793%
Apr2015	0.0668%	-1.3961%	1.4629%
May2015	3.2082%	0.3361%	2.8721%
Jun2015	0.7733%	0.9639%	-0.1906%
Jul2015	-0.1027%	-0.3136%	0.2109%
Aug2015	0.1156%	-0.1944%	0.3100%
Sep2015	-1.5526%	-0.0244%	-1.5282%
Oct2015	0.0325%	-0.5781%	0.6106%
Nov2015	-0.9758%	0.2993%	-1.2751%
Dec2015	-1.4117%	-0.4819%	-0.9298%
Jan2016	-4.0110%	-0.3826%	-3.6284%
Feb2016	-0.7112%	-1.5681%	0.8569%
Mar2016	-0.2873%	-0.0072%	-0.2801%
Apr2016	0.4900%	-0.4788%	0.9688%
May2016	1.4215%	0.1120%	1.3095%
Jun2016	-2.2771%	0.3726%	-2.6497%
Jul2016	-7.7826%	-0.8565%	-6.9261%
Aug2016	-0.2515%	-2.7137%	2.4622%
Sep2016	0.2972%	0.6352%	-0.3380%
Oct2016	-6.3625%	-0.7546%	-5.6079%
Nov2016	0.8238%	-2.4832%	3.3070%
Dec2016	0.4093%	1.4606%	-1.0513%
Jan2017	-0.9336%	-0.4018%	-0.5318%
Feb2017	1.0296%	0.0174%	1.0122%
Mar2017	-1.1915%	0.7879%	-1.9794%
Apr2017	2.3374%	-0.4911%	2.8285%
May2017	2.2685%	0.9361%	1.3324%
Jun2017	-0.9246%	0.0291%	-0.9537%
Jul2017	1.4415%	-0.8688%	2.3103%
Aug2017	-0.3391%	0.5801%	-0.9192%
Sep2017	2.9516%	-0.5358%	3.4874%
Oct2017	-1.0398%	1.5240%	-2.5638%
Nov2017	0.1135%	-0.7137%	0.8272%
Dec2017	1.4049%	0.0363%	1.3686%
Jan2018	3.0853%	0.1279%	2.9574%
Feb2018	0.9861%	0.6073%	0.3788%
Mar2018	0.1073%	0.4031%	-0.2958%
Apr2018	0.7342%	-0.0005%	0.7347%
May2018	-4.4219%	0.8401%	-5.2620%
Jun2018	-1.3152%	-2.132%	0.8168%
Jul2018	-0.9978%	0.1705%	-1.1683%
Aug2018	-2.1813%	-0.8028%	-1.3785%
Sep2018	1.4493%	-1.2024%	2.6517%
Oct2018	-0.4141%	0.7242%	-1.1383%
Nov2018	-0.8644%	-0.3299%	-0.5345%
Dec2018	-1.8463%	0.2816%	-2.1279%
Jan2019	1.8541%	-0.1484%	2.0025%
Feb2019	0.8874%	0.9624%	-0.0750%
Mar2019	1.1534%	-0.0384%	1.1918%
Apr2019	-1.0536%	0.7769%	-1.8305%
May2019	-1.3444%	-0.5643%	-0.7801%
Jun2019	-1.4101%	-0.0245%	-1.3856%
Jul2019	-1.7027%	-0.3716%	-1.3311%
Aug2019	-2.4451%	-0.2453%	-2.1998%
Sep2019	1.7041%	-0.4927%	2.1968%
Oct2019	2.3017%	0.8562%	1.4455%
Nov2019	1.7775%	0.4630%	1.3145%
Dec2019	1.7312%	0.1173%	1.6139%
Jan2020	-0.2520%	0.3170%	-0.5690%
Feb2020	-0.9451%	-0.3123%	-0.6328%

Source: Author's Computation

Table 10: Forecasting performance of rolling window forecasting method through MSE, MAE, MAPE and correct sign prediction criterion

	Performance value
MSE	0.0004058622
MAE	0.01590581
MAPE	223.4949
Correct sign	0.5697674

Source: Author's Computation

Conclusion

This paper uses the R software in determining the best ARMA model for predicting the growth rate of the exchange rate between the US dollar and a unit of the British pound (denoted as \$/£). The MSE, MAE, MAPE and correct sign prediction criterion are used to compare the forecasting performance of the rolling window forecasting method with the traditional forecasting method. The results obtained suggest that the traditional forecasting method seems to perform better judging by MSE, MAE and MAPE. In other words, the traditional forecasting method appears to perform better in predicting the magnitude (i.e., size) by which $\Delta \log (\$/\text{£})$ series changes overtime. However, the results also suggest that the rolling window forecasting method seems to perform better based on the correct sign prediction criterion. Hence, the rolling window forecasting method appears to perform better in predicting the changes in the sign of $\Delta \log (\$/\text{£})$ series.

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Appendices

Appendix A: ADF Φ_1 joint hypothesis test on unit root and drift

Test	H0	Stat	CV	Reject?	Conclusion
ADF	$\log(\$/\pounds)$ has a unit root and a drift	4.410	4.590	not reject	$\log(\$/\pounds)$ has a unit root and a drift
ADF	$\Delta\log(\$/\pounds)$ has no unit root and/or drift	56.444	4.590	reject	$\Delta\log(\$/\pounds)$ has no unit root and/or no drift

Appendix B: ADF Φ_1 joint hypothesis test on unit root and drift

1-step ahead	In-sample data used Rolling window	Best model selected by AIC	Ljung-Box Test (p-value)		
			Lag 20	Lag 30	Lag 50
Jan2013	Feb1971 - Dec2012	ARMA(2,3)	0.5005	0.3630	0.1578
Feb2013	Mar1971 - Jan2013	ARMA(2,3)	0.5063	0.3630	0.1635
Mar2013	Apr1971 - Feb2013	ARMA(4,5)	0.6532	0.4552	0.2531
Apr2013	May1971 - Mar2013	ARMA(2,3)	0.5121	0.3625	0.1658
May2013	Jun1971 - Apr2013	ARMA(2,3)	0.5135	0.3786	0.2019
Jun2013	Jul1971 - May2013	ARMA(2,3)	0.5203	0.3798	0.2014
Jul2013	Aug1971 - Jun2013	ARMA(2,3)	0.5325	0.3999	0.2237
Aug2013	Sep1971 - Jul2013	ARMA(2,3)	0.5290	0.4010	0.2150
Sep2013	Oct1971 - Aug2013	ARMA(2,3)	0.4900	0.4040	0.2240
Oct2013	Nov1971 - Sep2013	ARMA(2,3)	0.4780	0.3800	0.2100
Nov2013	Dec1971 - Oct2013	ARMA(2,3)	0.4910	0.3940	0.2290
Dec2013	Jan1972 - Nov2013	ARMA(2,3)	0.4980	0.3930	0.2290
Jan2014	Feb1972 - Dec2013	ARMA(2,3)	0.4869	0.3829	0.2230
Feb2014	Mar1972 - Jan2014	ARMA(2,3)	0.4934	0.3790	0.2254
Mar2014	Apr1972 - Feb2014	ARMA(2,3)	0.4892	0.3742	0.2249
Apr2014	May1972 - Mar2014	ARMA(2,3)	0.4900	0.3720	0.2200
May2014	Jun1972 - Apr2014	ARMA(2,3)	0.4906	0.3772	0.2212
Jun2014	Jul1972 - May2014	ARMA(2,3)	0.4918	0.3682	0.2160
Jul2014	Aug1972 - Jun2014	ARMA(2,3)	0.5777	0.4124	0.2114
Aug2014	Sep1972 - Jul2014	ARMA(2,3)	0.5846	0.4160	0.2130
Sep2014	Oct1972 - Aug2014	ARMA(2,3)	0.5977	0.4259	0.1964
Oct2014	Nov1972 - Sep2014	ARMA(2,3)	0.5564	0.4037	0.1827
Nov2014	Dec1972 - Oct2014	ARMA(2,3)	0.5617	0.4136	0.1843
Dec2014	Jan1973 - Nov2014	ARMA(2,3)	0.5522	0.3951	0.1758
Jan2015	Feb1973 - Dec2014	ARMA(2,3)	0.5508	0.3971	0.1754
Feb2015	Mar1973 - Jan2015	ARMA(4,5)	0.1414	0.3967	0.2432
Mar2015	Apr1973 - Feb2015	ARMA(2,3)	0.5324	0.3839	0.2110
Apr2015	May1973 - Mar2015	ARMA(0,1)	0.4386	0.4243	0.1345
May2015	Jun1973 - Apr2015	ARMA(4,5)	0.6953	0.5006	0.2947
Jun2015	Jul1973 - May2015	ARMA(0,1)	0.4734	0.4567	0.1530
Jul2015	Aug1973 - Jun2015	ARMA(2,3)	0.6118	0.4684	0.2696
Aug2015	Sep1973 - Jul2015	ARMA(3,3)	0.5219	0.3879	0.2108
Sep2015	Oct1973 - Aug2015	ARMA(2,3)	0.5990	0.4558	0.2584
Oct2015	Nov1973 - Sep2015	ARMA(2,3)	0.5910	0.4576	0.2608
Nov2015	Dec1973 - Oct2015	ARMA(2,3)	0.5755	0.4511	0.2741
Dec2015	Jan1974 - Nov2015	ARMA(2,3)	0.6130	0.4552	0.2987
Jan2016	Feb1974 - Dec2015	ARMA(2,3)	0.1636	0.3740	0.2520
Feb2016	Mar1974 - Jan2016	ARMA(2,3)	0.6479	0.4766	0.3100
Mar2016	Apr1974 - Feb2016	ARMA(2,3)	0.6448	0.4820	0.3036
Apr2016	May1974 - Mar2016	ARMA(2,3)	0.6420	0.4940	0.3150
May2016	Jun1974 - Apr2016	ARMA(2,3)	0.6300	0.4790	0.3088
Jun2016	Jul1974 - May2016	ARMA(0,1)	0.5100	0.4830	0.1830
Jul2016	Aug1974 - Jun2016	ARMA(2,3)	0.6896	0.5018	0.2960
Aug2016	Sep1974 - Jul2016	ARMA(0,1)	0.4987	0.4432	0.1930
Sep2016	Oct1974 - Aug2016	ARMA(2,3)	0.5723	0.4082	0.2990
Oct2016	Nov1974 - Sep2016	ARMA(2,3)	0.5725	0.4084	0.2944
Nov2016	Dec1974 - Oct2016	ARMA(0,1)	0.3980	0.3840	0.1800
Dec2016	Jan1975 - Nov2016	ARMA(4,1)	0.2410	0.3424	0.1920
Jan2017	Feb1975 - Dec2016	ARMA(0,1)	0.3928	0.3675	0.1678
Feb2017	Mar1975 - Jan2017	ARMA(2,3)	0.4790	0.3501	0.2640
Mar2017	Apr1975 - Feb2017	ARMA(2,3)	0.4600	0.3360	0.2547
Apr2017	May1975 - Mar2017	ARMA(2,3)	0.4844	0.3820	0.2901
May2017	Jun1975 - Apr2017	ARMA(2,3)	0.4730	0.3734	0.3066
Jun2017	Jul1975 - May2017	ARMA(2,3)	0.4483	0.3604	0.3109
Jul2017	Aug1975 - Jun2017	ARMA(2,3)	0.4034	0.3397	0.2857
Aug2017	Sep1975 - Jul2017	ARMA(2,3)	0.4271	0.3620	0.3340
Sep2017	Oct1975 - Aug2017	ARMA(4,5)	0.6302	0.4480	0.4215
Oct2017	Nov1975 - Sep2017	ARMA(2,3)	0.4777	0.4110	0.3650
Nov2017	Dec1975 - Oct2017	ARMA(4,5)	0.3920	0.3684	0.4020
Dec2017	Jan1976 - Nov2017	ARMA(2,3)	0.3483	0.3989	0.2654

App B Cont'd

Jan2018	Feb1976 - Dec2017	ARMA(2,3)	0.4706	0.3893	0.3428
Feb2018	Mar1976 - Jan2018	ARMA(2,3)	0.5329	0.4156	0.3660
Mar2018	Apr1976 - Feb2018	ARMA(2,3)	0.3650	0.3832	0.2931
Apr2018	May1976 - Mar2018	ARMA(0,1)	0.3341	0.3305	0.2200
May2018	Jun1976 - Apr2018	ARMA(4,5)	0.5499	0.3809	0.3847
Jun2018	Jul1976 - May2018	ARMA(2,3)	0.3170	0.3679	0.3083
Jul2018	Aug1976 - Jun2018	ARMA(2,3)	0.5026	0.3882	0.3522
Aug2018	Sep1976 - Jul2018	ARMA(2,3)	0.4827	0.3737	0.3499
Sep2018	Oct1976 - Aug2018	ARMA(2,3)	0.4722	0.334	0.3260
Oct2018	Nov1976 - Sep2018	ARMA(0,1)	0.3887	0.3547	0.2315
Nov2018	Dec1976 - Oct2018	ARMA(1,5)	0.3432	0.425	0.2840
Dec2018	Jan1977 - Nov2018	ARMA(1,5)	0.3550	0.4287	0.3030
Jan2019	Feb1977 - Dec2018	ARMA(2,3)	0.5000	0.325	0.3240
Feb2019	Mar1977 - Jan2019	ARMA(1,5)	0.3520	0.4097	0.2903
Mar2019	Apr1977 - Feb2019	ARMA(0,1)	0.3886	0.3333	0.2340
Apr2019	May1977 - Mar2019	ARMA(4,1)	0.3522	0.3700	0.2420
May2019	Jun1977 - Apr2019	ARMA(0,1)	0.3774	0.3268	0.2215
Jun2019	Jul1977 - May2019	ARMA(1,5)	0.3203	0.3827	0.2580
Jul2019	Aug1977 - Jun2019	ARMA(2,3)	0.4782	0.2952	0.2858
Aug2019	Sep1977 - Jul2019	ARMA(2,3)	0.4782	0.3014	0.3173
Sep2019	Oct1977 - Aug2019	ARMA(2,3)	0.4940	0.3010	0.3190
Oct2019	Nov1977 - Sep2019	ARMA(2,3)	0.5202	0.3193	0.3413
Nov2019	Dec1977 - Oct2019	ARMA(0,1)	0.3959	0.3249	0.2149
Dec2019	Jan1978 - Nov2019	ARMA(2,3)	0.4898	0.2867	0.2750
Jan2020	Feb1978 - Dec2019	ARMA(2,3)	0.5278	0.2791	0.2260
Feb2020	Mar1978 - Jan2020	ARMA(0,1)	0.4402	0.3080	0.1700

Source: Author's Comutation

Appendix C: Time series plot of residuals of models used in rolling window forecast

Feb1971 - Dec2012

Time series plot of the residuals

Mar1971 - Jan2013

Time series plot of the residuals

Apr1971 - Feb2013

Time series plot of the residuals

May1971 - Mar2013

Time series plot of the residuals

Jun1971 - Apr2013

Time series plot of the residuals

Jul1971 - May2013

Time series plot of the residuals

Feb1972 - Dec2013

Time series plot of the residuals

Mar1972 - Jan2014

Time series plot of the residuals

Apr1972 - Feb2014

Time series plot of the residuals

May1972 - Mar2014

Time series plot of the residuals

Jun1972 - Apr2014

Time series plot of the residuals

Jul1972 - May2014

Time series plot of the residuals

Feb1974 - Dec2015

ACF plot of the residuals

Mar1974 - Jan2016

ACF plot of the residuals

Apr1974 - Feb2016

ACF plot of the residuals

May1974 - Mar2016

ACF plot of the residuals

Jun1974 - Apr2016

ACF plot of the residuals

Jul1974 - May2016

ACF plot of the residuals

Aug1974 - Jun2016

ACF plot of the residuals

Sep1974 - Jul2016

ACF plot of the residuals

Oct1974 - Aug2016

ACF plot of the residuals

Nov1974 - Sep2016

ACF plot of the residuals

Dec1974 - Oct2016

ACF plot of the residuals

Jan1975 - Nov2016

ACF plot of the residuals

Feb1975 - Dec2016

ACF plot of the residuals

Mar1975 - Jan2017

ACF plot of the residuals

Apr1975 - Feb2017

ACF plot of the residuals

May1975 - Mar2017

ACF plot of the residuals

Jun1975 - Apr2017

ACF plot of the residuals

Jul1975 - May2017

ACF plot of the residuals

Aug1975 - Jun2017

ACF plot of the residuals

Sep1975 - Jul2017

ACF plot of the residuals

Oct1975 - Aug2017

ACF plot of the residuals

Nov1975 - Sep2017

ACF plot of the residuals

Dec1975 - Oct2017

ACF plot of the residuals

Jan1976 - Nov2017

ACF plot of the residuals

Feb1976 - Dec2017

ACF plot of the residuals

Mar1976 - Jan2018

ACF plot of the residuals

Apr1976 - Feb2018

ACF plot of the residuals

May1976 - Mar2018

ACF plot of the residuals

Jun1976 - Apr2018

ACF plot of the residuals

Jul1976 - May2018

ACF plot of the residuals

Aug1976 - Jun2018

ACF plot of the residuals

Sep1976 - Jul2018

ACF plot of the residuals

Oct1976 - Aug2018

ACF plot of the residuals