What is the Optimal Financial Structure Applicable to All Companies

Chimba Ulrich Djomo (a)

(a) University of Ngaoundéré, Cameroon

ABSTRACT

To maximize value, companies must adopt the optimal financial structure. However, identifying this structure is not easy. Literature does not give satisfactory results. To remedy this difficulty, we have sought the optimal financial structure applicable to all companies. Thus, in a new approach by the cost of capital, using modelling, we identify an optimal financial structure model that can neutralize the risk of default of debt: D/K with t ≃ i but t > i.

Introduction

The objective of corporate finance in the broad sense is to maximize shareholder wealth and in the narrow sense to maximize share price (Damodaran, 2006). This maximization requires the respect of financing principle which stipulates that the debt/equity ratio must be that which minimize the cost of capital while maximizing the value of the company. The aim is to identify the optimal financial structure. This financial structure is represented by the ratio D/K.

In view to identify this optimal financial structure, our method is mathematical modelling. We will use mathematical tools to develop a formula that can be retained as the optimal financial structure model.

Our study starts with literature; followed by methodology; findings; verifications; discussions; and ends with the possibility of a putting in practice.

In literature review, we present the two currents of thought that exist on the optimal financial structure, as well as the problem born of their confrontation. At research and methodology, we study the function of the cost of capital explained by D/K. In findings, we present the formula of our model of optimal financial structure which is then verified through verifications. Our discussions have two components: contributions and limits. For practice, we give means to apply the optimal financial structure model and an example of an old company which names SHELL.

Without wanting to say everything here, let’s start with the literature review in the following line.

Literature Review

Research on the optimal financial structure has traditionally opposed two streams of thought: the first which maintains that there is no optimal financial structure and the second which supports the contrary.
The first stream was proposed and developed by Modigliani and Miller in 1958 in their work on the neutrality of debt in relation to the value of the company. The improbable assumptions they have adopted (no taxes, no risk of default, no transaction fees, etc.) have given them a lot of criticism. In spite of these criticisms, Miller (1979) will reaffirm the neutrality of the debt after taking into account taxes (companies and individuals) and the costs of bankruptcies in a new study.

The second stream which affirms the existence of the optimal financial structure proposes five approaches.

The first, which is the oldest, is the comparative analysis where the debt ratio of a company is compared to the average debt ratio of its sector of activity. However, companies in the same sector can differ significantly in terms of size, tax rates, risk and growth.

The second is based on the operating profit. It consists of determining the maximum probability of bankruptcy that a company is likely to accept and setting the amounts of debt accordingly. This approach ignores the appreciation of disciplinary power, agency costs and the loss of flexibility induced by debt.

The third is the cost of capital approach. It consists of estimating the cost of debt and the cost of equity for different debt ratios of a company, and using these costs to calculate the cost of capital in order to determine the debt ratio which leads to the lowest cost of capital. This minimum cost of capital maximizes the value of the company. The limit of this approach is to assume that the operating result is never affected by the risk of debt. When this is not the case, minimizing the cost of capital no longer likely to maximize the value of the company and the optimal financial structure is obtained by the approach that comes.

The fourth is the adjusted present value approach. It starts with the determination of the value of the company without debt. Debt is gradually added and the net effect on the value of the company is measured by the arbitratio of the advantages and disadvantages. Then, the value of the company is estimated for different debt ratios. The debt ratio that maximizes the value of the company is the optimal financial structure. This approach does not take in account the estimation of the probability of bankruptcy.

The fifth is the yield differential approach, which estimates that the optimal financial structure is the debt ratio for which the difference between the return on equity and the cost of equity is maximized. This approach is based on the analysis of good investments and presents the danger of driving the company to sub-invest.

Confrontations arise between these two currents of thought: the first current of thought criticizes the second for affirming the existence of the optimal financial structure without identifying it, and the second current of thought also criticizes the first to be based on implausible hypotheses.

What is the optimal financial structure applicable to all companies?

Once we think that there is an optimal financial structure, we are now looking for that one which is applied to all companies.

Research and Methodology

Our method is for all the assumptions of the optimal financial structure approach by minimizing the cost of capital. Knowing that the cost of capital is made up of the cost of equity, cost of the debt, and the weighting of the market values of equity and debt: our study will estimate the cost of capital according to the ratio \( \frac{D}{K} \), and will study the function obtained to search extreme minimums.

However, let us first make a few reminders:

Reminder 1: the cost of equity will always be higher than the cost of debt (\( t > i \))

The shareholders who own the company bear a residual risk greater than the risk of the lenders and consequently will always require a higher remuneration than the lenders.

Reminder 2: if \( D > K \) then \( \frac{D}{K} \) belongs to \([0; +\infty [\)

Naturally, \( D \geq 0 \) and \( K > 0 \) (because if \( K = 0 \) the company does not exist) so, \( \frac{D}{K} \geq 0 \).

If a company issue a borrowing guaranteed by an independent, infallible, much safer and more important entity than companies or banks (such as a government or states cooperative ; Currently in European Union, many states as France have guaranteed debts of companies), if this borrowing is a bond redeemable for shares (automatically convertible into shares at a specific date before maturity), this company has the possibility to borrow an indeterminate amount: infinity (In mathematics, an indeterminate increase amount refers to \( +\infty \)). Its debt can also tend toward infinity (Before any new borrowing, the amount of old debt in the assessment is known [constant]; in mathematics, \(+\infty [\text{borrowing}] + \text{constant [old debt]} = +\infty [\text{new debt}]\). The financial tradition proposes to companies a ratio \( \frac{D}{K} < 1 \) in order to avoid any default of payment. But if a company have two or three subsidiaries in growing, and decides to issue a bond redeemable for shares of its subsidiaries, this company will be able to avoid any default while maintaining a high ratio \( \frac{D}{K} < 1 \). By this technique, we have neutralized the default on the payment of the debt as well as any dilution of the capital).
If debt can tend towards infinity, the ratio $\frac{D}{K}$ will also tend towards infinity. Because the infinity divided by a constant (K) always gives infinity (Before any new borrowing, the amount of equity is known). So $\frac{D}{K}$ belongs to $[0; +\infty[$.

Reminder 3: the cost of equity and the cost of debt are mathematic parameters

When we study an element according to one of its components, the other components behave like parameters. $\frac{D}{K}$ represents the weighting, so the cost of equity and the cost of debt are mathematic parameters.

Definitions and Study

$$\text{tc} = \frac{Kt + Di}{K + D}$$

$t$: cost of capital

$i$: cost of debt (after tax)

$K$: market value of equity

$D$: market value of debt

Naturally $\text{tc} \geq 0$, in the absence of capital rationing the upper limit of $\text{tc}$ is indeterminate. So $\text{tc}$ belongs to $[0; +\infty[$.

Let us say $\text{tc}$ according to $\frac{D}{K}$

$$\text{tc} = \frac{Kt + Di}{K + D}$$

(1)

Let us call “X” the ratio $\frac{D}{K}$, $X = \frac{D}{K}$ so, $D = KX$

(2)

(2) in (1) : \[\text{tc} = \frac{Kt + KXi}{K + KX} = \frac{tX + i}{X + 1}\]

(3)

We estimate $\text{tc} = \frac{X_i + t}{X + 1}$ with $X = \frac{D}{K}$.

Representation

Call $h: R+ \rightarrow R+: X \rightarrow h(X) = \frac{X_i + t}{X + 1}$

$h(X)$ is the cost of capital function defined by the ratio $\frac{D}{K}$.

Limits

$$\lim_{X \to 0} h(x) = h(0) = t$$

$$\lim_{X \to +\infty} h(x) = \lim_{X \to +\infty} \frac{X_i + t}{X + 1} = \lim_{X \to +\infty} \frac{X_i}{X} = i$$

$h(X)$ has a horizontal asymptote (line) of equation $y = i$, because $i$ belongs to $R+$.

Differentiability

$h(X)$ is a rational function naturally differentiable on its domain ($Df$).

$$h'(X) = \frac{(xi + t)'(x + 1) - (x + 1)'(xi + t)}{(x + 1)^2}$$

$$= \frac{x(x + 1) - 1}{(x + 1)^2}$$

$t$ and $i$ are constants their differentiability is zero.

$$= \frac{i + t}{(x + 1)^2}$$

Naturally $t$ and $i$ are positive, we know that $t > i$ so $t > 0$ and $i > 0$. Whatever $X$ belonging to $[0; +\infty[$, $(X + 1)^2 > 0$, so $\frac{1}{(x + 1)^2} > 0$ and $(i-t) \times \frac{1}{(x + 1)^2} < 0$

Consequently, $h'(X) < 0$ for any $X$ belonging to $[0; +\infty[$.
The curve of \( h(X) \) is decreasing on \( [0; +\infty[ \). 

### Table 1: Board of differentiability

<table>
<thead>
<tr>
<th>( X )</th>
<th>( 0 )</th>
<th>( +\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h'(X) )</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>( h(X) )</td>
<td>( t )</td>
<td>( i )</td>
</tr>
</tbody>
</table>

Note: ourselves

### Graphic

In order to conform to the rules of the vector space (benchmarks), we assign known values to \( i \) and \( t \).

Choose \( i = 1 \) and \( t = 2 \)

![Figure 1: Curve of \( h(X) \) Choosing \( i = 1 \) and \( t = 2 \)](image)

Note: ourselves

### Findings

The previous analysis shows us that the cost of capital curve is strictly decreasing, and its minimum value is close to \( i \) as indicate its horizontal asymptote of equation \( y = i \).

So, when \( t_c \) varies according to \( \frac{D}{K} \), its minimum is obtained when \( t_c \approx i \) \( \quad (4) \)

However, we know that \( t_c = \frac{Kt + Di}{K + D} \) \( \quad (1) \).

(1) in (4) \( \iff i \approx \frac{Kt + Di}{K + D} \)
\( \iff Ki + Di \approx Ki + Di \)
\( \iff Ki \approx Ki \)
\( \iff t \approx i \) \( \quad (5) \)

Knowing that \( t > i \), we also discover that \( i \approx t \) when \( t_c \) is minimum.

So, the optimal financial structure model is:

\[ \frac{D}{K} \text{ with } t \approx i \text{ but } t > i. \]

### Verifications

This is our model:

\[ \frac{D}{K} \text{ with } t \approx i \text{ but } t > i. \]

\( t \) and \( i \) are mathematical parameters, so we can give them precise values.

Example: \( t \% = 9 \) and \( i \% = 7 \) \( \iff (t\%; i\%) \in (9; 7) \). Let us say: \( K = 10 \) and \( D = 5 \)
If we add debt, how will move tc and D+K (value of company)? Request to this question is given in the board below.

**Table 2:** tc is minimal when the value of the company (D+K) is maximal, and tc cannot never equal i or go under it.

<table>
<thead>
<tr>
<th>possibilities</th>
<th>Borrowing</th>
<th>D</th>
<th>K</th>
<th>D/K</th>
<th>j%</th>
<th>t%</th>
<th>tc%</th>
<th>D+K</th>
</tr>
</thead>
<tbody>
<tr>
<td>today</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>5/10</td>
<td>7</td>
<td>9</td>
<td>8,33</td>
<td>15</td>
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<tr>
<td>1st</td>
<td>1</td>
<td>6</td>
<td>10</td>
<td>6/10</td>
<td>7</td>
<td>9</td>
<td>8,25</td>
<td>16</td>
</tr>
<tr>
<td>2nd</td>
<td>2</td>
<td>7</td>
<td>10</td>
<td>7/10</td>
<td>7</td>
<td>9</td>
<td>8,17</td>
<td>17</td>
</tr>
<tr>
<td>3rd</td>
<td>3</td>
<td>8</td>
<td>10</td>
<td>8/10</td>
<td>7</td>
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<td>10</td>
<td>10</td>
<td>1</td>
<td>7</td>
<td>9</td>
<td>8</td>
<td>20</td>
</tr>
</tbody>
</table>

Note: ourselves

When we add debt (D), the value of the company (K+D) adds consequently. The higher the value of the company, lower will be the cost of its capital. That cost of capital approaches of the cost of debt when the value of the company increases. This cost of capital cannot never equal cost of debt or go under it. So \( tc \simeq i \leftrightarrow t \simeq i \) (We have demonstrated it up before), when we bring together cost of debt and cost of equity, the minimal cost of capital is found easily and rapidly. So \( \frac{D}{K} \) with \( t \simeq i \) but \( t > i \), could be identified as the optimal financial structure of all companies.

**Discussions**

Our discussion is divided into contributions and limits.

**Contributions**

Contrary to the first stream of thought, our study affirms the existence of an optimal financial structure and proposes a model. Unlike the previous approach by the cost of capital which asserts that the optimal financial structure is found in the volumes of debt and equity, we affirm that the optimal financial structure is rather found in the reconciliation of the cost of equity with the cost of debt and in the two illustrated relations between these two costs. Our model values the support of government in the indebtedness of companies for which it stands surety. This opens up the prospect of neutralizing the risk of default which moves from the firm’s assets to an infallible bail. The main obstacle to a company’s debt is the risk of default. By the state guarantee, and bonds redeemable for shares, we neutralize this risk and project the company’s indebtedness to infinity.

Contrary to the financial tradition which conditions companies to keep a ratio \( \frac{D}{K} < 1 \) in order to avoid any default of payment; we affirm that a company can have a ratio \( \frac{D}{K} > 1 \) without any default of payment. The risk of default of payment can be neutralized with a bond redeemable for shares. A company in its mature life phase, with a subsidiary in growing, can significantly increase its debt by issuing bonds redeemable for shares of its subsidiary; its debt will be high with a low risk of bankruptcy. Or, that same company can have two or three subsidiaries in growing, and decides to issue a bond redeemable for shares of its subsidiaries.

**Limits**

The main limit of the proposed model is in the assumptions associated with any approach by the cost of capital in maximization of the value of the company. To this limit we can also add controversy over bond redeemable for shares. After assessing the cost of equity, the company determines the amount of additional debt that will bring the total cost of debt closer to the cost of equity.

Justification for reconciling the cost of debt with the cost of equity

When a company decides to use its additional borrowing capacity, it is prepared to bear a very high risk of default. The financial situation of this company in relation to bankruptcy is assessed from the financial charges related to the additional indebtedness and also from the financial charges that existed before. These financial charges can be huge to the point of making the EBIT negative, and thus loose tax deduction (zero rate). If there is no tax benefit, the cost of the debt remains the pré-tax cost. This high cost of debt could approximates the cost of equity.

Stages of companies’ life and optimal financial structure

Traditionally companies adopt their optimal financial structure during the stage of stable growth.
Shell case

Shell is a company that has existed for more than one century (since 1890). Now, if Shell decides to adopt its optimal financial structure in the next year by taking a borrowing that will be automatically convertible into shares at a specific maturity (a date before the maturity of the principal of the borrowing), if we assume that its debt ratio is \( \frac{D}{K} = \frac{80}{140} \) (US Billions)

Assuming also that Shell’s cost of equity is 11.5% and its cost of debt is 8%, its future borrowing in the next year must bring its cost of debt to 13% while allowing its costs of equity to remain almost greater than 13%.

NB: the difference (absolute value) between the value of \( t \) and the value \( i \) must be strictly less than 1%: \( |t - i| < 0.001\% \). Non-observance is a bad bringing together.

Institutional Review Board Statement: Ethical review and approval were waived for this study, due to that the research does not deal with vulnerable groups or sensitive issues.

Data Availability Statement: The data presented in this study are available on request from the corresponding author. The data are not publicly available due to privacy.

Conflicts of Interest: The author declares no conflict of interest.

References


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